International Journal of Swarm Intelligence Research
Volume 14 • Issue 3

A Many-Objective Evolutionary Algorithm Based on Non-Linear Dominance

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ABSTRACT

With the increase in the number of objectives, the number of non-dominated solutions will also increase sharply. The sorting method based on the traditional Pareto dominance is not sufficiently distinguishable from the solutions and cannot provide enough selection pressure when the population size is small. In this article, a new non-linear dominance (NLD) method is proposed. The main motivation of this method is from the perspective of storage solutions. The number of solutions is small and the difference between each component is as large as possible, so the part of the first quadrant, the second, and the fourth quadrant near the first quadrant becomes the dominant interval, except for the distance too far also defined as the dominant interval, for which construct a parabolic shape of the non-dominant interval. Based on this relationship, the authors propose a non-linear dominated many-objective evolutionary algorithm (NLDEA), which can solve the irregular Pareto front. Experiments show that NLDEA is competitive with the most advanced methods for various scalable benchmark problems.

KEYWORDS

Evolutionary Algorithm, Many-Objective Optimization, Mating Selection, Non-Linear Dominance, Pareto Dominance

INTRODUCTION

Many-objective optimization problems (MaOPs), which are problems requiring optimization of more than three conflicting objectives, have recently attracted widespread interest in the evolutionary multi-objective optimization (EMO) community. NSGA-II (Deb et al., 2002), one of the most well known EMO methods based on the principle of Pareto dominance selection, has been used to solve various multi-objective optimization problems (MOPs). It has achieved great success in solving various MOPs, including (Lu et

DOI: 10.4018/IJSIR.323422

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al., 2020; S. Zhu & Xu, 2018) a wide range of application cases. The concept of Pareto dominance, an intuitive and qualitative notion of compromise, has been commonly adopted to distinguish the quality of solutions for traditional two- or three-dimensional MOPs. However, the effectiveness of Pareto-based multi-objective evolutionary algorithms in solving multi-objective evolutionary problems has dramatically decreased. The main challenge of these methods is the loss of Pareto-based selection pressure toward the true Pareto front (PF) while the number of objectives M grows (Santos & Takahashi, 2016), i.e., the solutions become incomparable due to dominance resistance, and the difficulty of balancing convergence with diversity (Deb & Jain, 2013). To address this problem, various methods to enhance Pareto dominance have been proposed to solve MaOPs, which can be broadly classified into two categories.

The first category is concerned with developing new dominance relationships, and the basic idea is to increase the probability that two candidate solutions on MaOP are comparable. In the existing literature there are many approaches to developing new dominance relations, such as expanding the dominance region (J. Liu et al., 2022; Sato et al., 2007; C. Zhu et al., 2015; S. Zhu et al., 2022), latticing the object space (Laumanns et al., 2002; Yang et al., 2013), using fuzzy logic (Wang & Jiang, 2007; He et al., 2013; Qasim et al., 2022), defining dominance relations with weight vectors (Yuan et al., 2016; Elarbi et al., 2017), etc.

The second category is characterized by a combination of Pareto dominance and additional selection criteria. This method first uses Pareto dominance to eliminate a few poorer candidate solutions, and then uses a quadratic selection criterion to distinguish the non-comparable candidates. Existing methods of this type use three main basic ideas. The first idea is to create new criteria to bias the criteria with better convergence and diversity among the non-comparable candidate solutions, such as KnEA (Zhang et al., 2014), VaEA (Xiang et al., 2017), and AGEMOEA-II (Panichella, 2022). The second idea uses performance metrics to distinguish the quality of non-comparable candidate solutions by selecting the candidate solutions with higher contribution, such as HypE (Bader & Zitzler, 2011) and ARMOEA (Tian et al., 2017a). The third idea is to combine Pareto dominance with decomposition-based algorithms such as MOEA/DD (Li et al., 2014) and FDEA (S. Liu et al., 2016).

Existing modified Pareto dominance criteria, also known as relaxed Pareto dominance relationships, such as α-dominance (Ikeda et al., 2001), CDAS (Sato et al., 2007), CNα-dominance (J. Liu et al., 2019), Cα-dominance (J. Liu et al., 2022), generalized Pareto optimal (GPO) (C. Zhu et al., 2015), and ($M-1$)-GPD (S. Zhu et al., 2021), to better distinguish solutions and select elite solutions with enhanced selection pressure. Most of them, however, have difficulty in maintaining the delicate balance between convergence and diversity. Excessive selection pressure usually tends to lead to deterioration of diversity maintenance, which may result in population aggregation to a sub region (or several small sub regions) of PF, while too little selection pressure leads to a decrease in convergence performance. Moreover, they are all linear, which is not effective enough for irregular PF problems. Pareto dominance decreases the ability to distinguish solutions on high-dimensional objective problems and lacks convergence performance, but is perhaps a good choice for maintaining diversity aspects. Therefore, this paper proposes a new dominance relation, nonlinear dominance (NLD), which can handle irregular PF problems well by initially screening out individuals with good convergence performance by expanding the dominance region, and then gradually shrinking the dominance region to maintain diversity as the population iterates.

In summary, the contributions of this paper are as follows:

1. A new dominance relation NLD is proposed, i.e., dominance relation without reference vector as a selection criterion, which can maintain a good balance of convergence and diversity. It can also be easily embedded into other existing MaOEAs, such as NSGA-III.
2. The proposed NLD scheme is almost parameter-less and is used in a novel MaOEA, i.e., NLD-based optimization, or NLDEA for short. NSGA-III_NLD and NLDEA are compared with other algorithms and tested in two test suites, including WFG, and MAF test suites. The effectiveness of NLD and the performance of NLDEA are verified by comparing them with several state-of-the-art MOEAs.
3. The superior performance of NLDEA has also been confirmed on real-world problems with irregular Pareto fronts (PFs).

RELATED WORK

The Pareto dominance relation is widely used in MOEA to distinguish the quality of candidate solutions. For the minimization MOP, a candidate solution **x** is said to Pareto dominate another solution **y**, denoted as **f (x) ≺ f (y)**, if and only if:

$$\begin{cases} \forall i \in 1,\ldots,M : f_i\left(x\right) \le f_i\left(y\right) \\ \exists j \in 1,\ldots,M : f_j\left(x\right) < f_j\left(y\right) \end{cases} \tag{1}$$

where $f\left(x\right) = \left(f_1\left(x\right), f_2\left(x\right), \ldots, f_M\left(x\right)\right)$ is the objective value of **x** and M is the number of objectives. If **x** is not better than **y,** and **y** is not better than **x**, then the two candidate solutions are not comparable or non-dominated with each other.

The α-dominance method (Ikeda et al., 2001) is representative of the extended solution dominance region method, which expands the dominance region of the candidate solution **x** by modifying the objective value:

$$\begin{cases} \forall i \in 1,\ldots,M : f_i\left(x\right) + \sum_{k\neq i} \alpha_i f_k\left(x\right) \le f_i\left(y\right) + \sum_{k\neq i} \alpha_i f_k\left(y\right) \\ \exists j \in 1,\ldots,M : f_j\left(x\right) + \sum_{k\neq j} \alpha_j f_k\left(x\right) < f_j\left(y\right) + \sum_{k\neq j} \alpha_j f_k\left(y\right) \end{cases} \tag{2}$$

where α_i is a hyper parameter.

CDAS (Sato et al., 2007) also shows the great potential of solving MaOPs:

$$f_i'\left(x\right) = \frac{\| f\left(x\right) \| \sin\left(\omega_i + S \cdot \pi\right)}{\sin\left(S \cdot \pi\right)}, i = 1,\ldots,M \tag{3}$$

where $f_i'\left(x\right)$ is the i th objective value after **x** is modified, $\| \cdot \|$ is a 2-parameter number, ω_i is the deferential angle between **x** and the i th axis, and $S \in \left[0.25, 0.5\right]$ is the parameter controlling the expansion.

The GPO and α-dominance are actually the same, but the value of the parameter α is set differently. In GPO (C. Zhu et al., 2015), one candidate solution **x** is superior to another candidate solution **y** when and only when the following conditions hold:

$$\begin{cases} \forall i \in 1,\ldots,M : f_i\left(x\right) - f_i\left(y\right) \le \sum_{k\neq i} \delta_i \left(f_k\left(y\right) - f_k\left(x\right)\right) \\ \exists j \in 1,\ldots,M : f_j\left(x\right) - f_j\left(y\right) < \sum_{k\neq j} \delta_j \left(f_k\left(y\right) - f_k\left(x\right)\right) \end{cases} \tag{4}$$

where $\delta_i = \sqrt{M-1} \cdot \tan\phi_i / \left(M-1\right)$, ϕ_i is the parameter of the expansion angle on the i th objective. The (M – 1)-GPD is similar to GPO, except that one objective is guaranteed to be constant at a time and the other objectives are expanded, as can be seen in (S. Zhu et al., 2021).

METHODOLOGY

The above methods can provide greater selection pressure by further expanding the dominant region, however, none of them can maintain the overall population diversity well. Therefore, this section describes the new dominance relation called "nonlinear domination."

Proposed Dominance Relation

In the proposed NLD method, NLD makes the first quadrant, a part of the second quadrant, and the fourth quadrant also into the dominance space, which ensures that each component of the understanding differs as much as possible and improves the diversity of the population. NLD also defines distant individuals, as the dominated relationship improves the convergence of the population, so that a parabolic shape of the dominance space appears. In addition, the coefficients of the parabola are dynamic with the number of iterations, maintaining a balance of convergence and diversity. The NLD is defined as:

$$\begin{cases} \forall i \in 1,\ldots,M : f_i\left(x\right) + \sum_{k \neq i} \gamma_i f_k^2\left(x\right) \leq f_i\left(y\right) + \sum_{k \neq i} \gamma_i f_k^2\left(y\right) \\ \exists j \in 1,\ldots,M : f_j\left(x\right) + \sum_{k \neq j} \gamma_j f_k^2\left(x\right) < f_j\left(y\right) + \sum_{k \neq j} \gamma_j f_k^2\left(y\right) \end{cases} \tag{5}$$

where $\gamma_i = 1 / \left(1 + 100 \exp\left(\alpha M \frac{t}{T}\right)\right)$ and α is the factor that controls convergence and diversity, M is the number of objectives, t is the number of current iterations, and T is the total number of iterations, respectively. For solution **x** and the solution **y**, if it satisfies the above definition, it is called **x** nld-dominance **y** and is denoted as $f\left(x\right) \prec_{nld} f\left(y\right)$. As can be seen from the Figure 1, NLD compared to other modified Pareto domination, the linear dominance relation seems to be difficult to work for the MOP problem of irregular PFs.

General Framework

The proposed NLDEA algorithm first generates a population by random initialization, calculates the objective value for each individual, and then performs evolutionary operators, including mate selection and reproduction operations (usually crossover and mutation) to generate progeny solutions. In this study, the dominant parent is selected for recombination by binary race selection, the reproduction operation uses simulated binary crossover (SBX) (Deb, 2011) and polynomial mutation (PM) (Deb & Goyal, 1996, and the environmental selection stage uses a nonlinear dominance ranking method to select superior individuals; the above steps are repeated until the termination condition is satisfied. The algorithmic framework is shown in Algorithm 1 and the Algorithm flow chart is shown in Figure 2.

Figure 1. Dominating areas obtained by five different relaxed Pareto-dominance relations in the bi-objective space. (a) Pareto dominance, (b) α-dominance, (c) (M-1)-GPD. (d) CDAS, (e) NLD

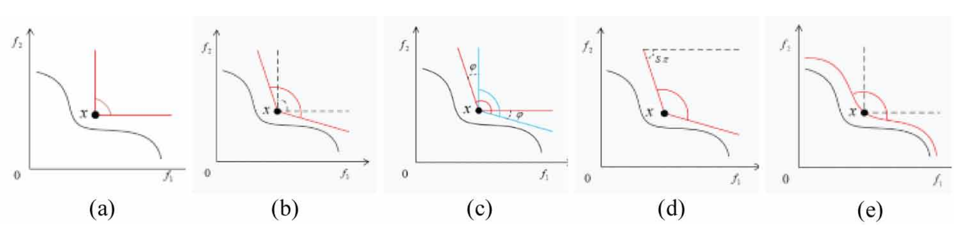
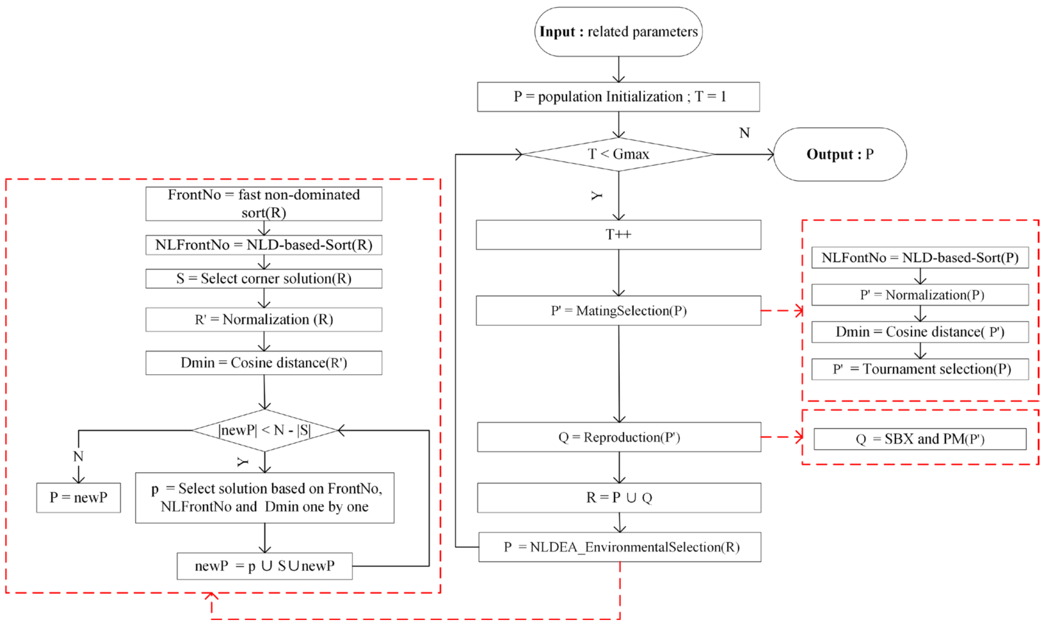


Figure 2. The algorithm flow chart of NLDEA



Algorithm 1. General Framework of NLDEA

Input: impact factor α, objectives number M, population size N and total iteration number T;
Output: survive population P
P = Population_initialization();
Evaluate population P;
$t = 1$;
while *termination condition not met* **do**
$t = t + 1$;
$\lambda = 1 \Big/ \left(1 + 100\exp\left(\alpha M \dfrac{t}{T}\right)\right)$; // where λ is the quadratic term coefficient of the parabola
P' = MatingSelection(P, λ);
Q = Reproduction (P');
$R = P \cup Q$;
P = NLDEA_EnvironmentalSelection(R, λ, N);
end
return P;

Objective Normalization

Normalizing the objective values of all individuals to a uniform scale is crucial in MaOPs. Within a MaOP, the scale of objectives may vary considerably and, therefore, without normalization, the distance measure defined in the objective space to maintain population diversity may be dominated by some objectives (Deb et al., 2010). In this regard, objective normalization is necessary to ensure a more reliable distance measure. Given a solution **x** with each of its objectives $f_i\left(x\right), i = 1, 2, \ldots, M$, normalize as follows:

$$f_i^{'}\left(x\right) = \frac{f_i\left(x\right) - z_i^*}{z_i^{nad} - z_i^*}, i = 1, 2, \ldots, M \tag{6}$$

where z^* and z^{nad} are the ideal point and the nadir point, respectively. Since the shape of PF is unknown at the time, z^* and z^{nad} are usually estimated from the current population. In this paper, $z^* = \left(z_1^*, z_2^*, \ldots, z_M^*\right)$ is the current minimum value for each objective and z^{nad} is approximated by *M* corner individuals as in (Liang et al., 2019). The corner individual, noted as x_i^c, is the individual whose corresponding objective vector is closest to objective axis i, i.e.:

$$x_i^c = \left\{x \arg\min dist^{\perp}\left(f\left(x\right), e^i\right), x \in P, i = 1, 2, \ldots, M\right\} \tag{7}$$

where $dist^{\perp}\left(f\left(x\right), e^i\right)$ denotes the vertical Euclidean distance between the objective vector $f\left(x\right)$, $f\left(x\right) = \left(f_1\left(x\right), f_2\left(x\right), \ldots, f_M\left(x\right)\right)$ is the objective value of solution x and M is the number of objectives, and the vector in the direction of the i th axis e^i. The nadir point z^{nad} can be defined as:

$$z_i^{nad} = f_i\left(x_i^c\right), i = 1, \ldots, M \tag{8}$$

where $f_i\left(x_i^c\right)$ is the i th objective value of the corner individual associated with the i th axis.

Mating Selection Strategy

During the mating selection process, the convergence performance of the solutions is evaluated using the modified ASF. The ASF values of solution i are calculated as:

$$ASF\left(i, w^i\right) = \max_{k=1:M}\left\{\left(f_k^i - z_k^*\right) / w_k^i\right\} \tag{9}$$

where z_k^* is the best value found by all current individuals on the objective k, f_k^i is the objective value of the solution i at the k th objective, w^i is the weight vector (J. Cheng et al., 2015) corresponding to solution i, and for the k th element w_k^i have:

$$w_k^i = \frac{f_k^i}{\sum_{l=1:M} f_l^i} \tag{10}$$

where its value will be set to 10^{-6} when w_k^i equals zero. At this point, the ASF value reflects the best degree of convergence of each solution. The smaller the ASF value of solution i, the better the convergence performance. In the literature, this ASF value is widely used for the convergence metric (Deb & Jain, 2013; He & Yen, 2015, 2016).

For the distance metric, this paper uses the cosine distance (M. Wang et al., 2022) to compute the dissimilarity between vectors. Given two solution vectors **x** and **y**, their cosine distances are:

$$d_{\cos}\left(x,y\right)=1-\frac{F'\left(x\right)\cdot F'\left(y\right)}{\| F'\left(x\right)\|_2 \times \| F'\left(y\right)\|_2} \tag{11}$$

where $F'\left(x\right)=\left(f_1'\left(x\right),f_2'\left(x\right),\ldots,f_M'\left(x\right)\right)$ is the normalized objective vector corresponding to solution **x**. For an individual **x,** the minimum distance metric is:

$$d_{\min}\left(x\right)=\min_{y\in P,y\neq x} d_{\cos}\left(x,y\right) \tag{12}$$

where P represents the population, **y** is a solution other than **x**, and a larger value of $d_{\min}\left(x\right)$ indicates a better diversity performance of solution **x**. The algorithm for the mating selection strategy is shown in Algorithm 2.

Environmental Selection Strategy

The need to focus on both convergence and diversity in environment selection is an extremely important part of the algorithm. In this paper, firstly, the coarse selection is performed by using fast non-dominated sorting on the original objective, followed by the whole NLD environment selection in three steps. First, the corner solution set $P_c\left(\left|P_c\right|\leqslant M\right)$ is added to the surviving solution set *S*. Second, the sorting result $1\times 2N$ matrix $NLFrontNo$ is obtained using non-linear domination. Third, $N-\left|P_c\right|$ solutions are filtered according to the sorting result, where N is the size of the retained population.

NLD fast non-dominated sort (NLD-Based-Sort), which refers to the use of the formula 13 to obtain the new objective, after which the new objective $\left(\widehat{f_1},\widehat{f_2},\ldots,\widehat{f_M}\right)$ of the population is executed as a fast-non-dominated sort operation to obtain the new dominated result:

$$\begin{bmatrix}\widehat{f_1}\\ \widehat{f_2}\\ \vdots\\ \widehat{f_{M-1}}\\ \widehat{f_M}\end{bmatrix}=\begin{bmatrix}1 & \lambda & \cdots & \lambda & \lambda\\ \lambda & 1 & \ldots & \lambda & \lambda\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \lambda & \lambda & \ldots & 1 & \lambda\\ \lambda & \lambda & \ldots & \lambda & 1\end{bmatrix}\cdot\begin{bmatrix}f_1^2\\ f_2^2\\ \vdots\\ f_{M-1}^2\\ f_M^2\end{bmatrix} \tag{13}$$

Algorithm 2. Mating Selection

Input: population P, population size N, the quadratic term coefficient λ
Output: parent population P'
ASF value calculation: calculate ASF value of each solution;
$NLFrontNo$ = NLD-based-Sort(P, λ);
Normalization(P);
Minimum Cosine distance calculation d_{min}: calculate minimum Cosine distance of each solution with others in population;
$P' = \varnothing$;
while $\left\|P'\right\| < N$ **do**
Two solutions p_1 and p_2 are randomly selected from the population P;
if $NLFrontNo(p_1) < NLFrontNo(p_2)$ *AND* $d_{min}(p_1) > d_{min}(p_2)$ **then**
$P' = P' \cup p_1$;
else if $NLFrontNo(p_1) > NLFrontNo(p_2)$ *AND* $d_{min}(p_1) < d_{min}(p_2)$ **then**
$P' = P' \cup p_2$;
else
if $ASF(p_1) < ASF(p_2)$ **then**
$P' = P' \cup p_1$;
else
$P' = P' \cup p_2$;
end
end
end
return P';

where $\lambda = 1 / \left(1 + 100 \exp\left(\alpha M \frac{t}{T}\right)\right)$, and α is the parameter that balances convergence and diversity, M is the number of objectives, t is the number of current iterations, and T is the total number of iterations. Environment selection pseudo code is shown in Algorithm 3.

Computational Complexity Analysis

During each iteration, the time complexity of the algorithm is mainly influenced by mating selection and environment selection, assuming a population size is N and objective dimension is M. In mating selection, the cosine distance calculation and ASF value calculation require $O\left(MN^2\right)$ and

Algorithm 3. NLDEA_Environmental Selection

Input: population size N, combined population R, the quadratic term coefficient λ
Output: survive population P
$P = \varnothing$ and excute fast non-dominated sorting on R;
$NLFrontNo$ = NLD-based-Sort(R, λ);
determinant corner solution set S which has minimum perpendicular Euclidean distance with direction vector of the i-th axis, $i = 1, 2, ..M$; // corner individuals are non-dominated
Normalization();
$P = P \cup S$, $R = R \backslash S$;
while $\left\lvert P \right\rvert < N$ **do**
Let R_{nd} be the set of all non-dominated solutions in R;
foreach $\mathbf{x} \in R_{nd}$ **do**
$d_{min}(\mathbf{x}) = \min_{y \in p} d_{cos}(\mathbf{x}, \mathbf{y})$
end
Sort individuals of R_{nd} in descending order of d_{min} values;
Let R_t is the top ranked N_t individuals in R_{nd}; // here $N_t = \min(0.5\ R, R_{nd})$
$Q = \{\mathbf{x} \in R_t, NLFrontNo(\mathbf{x}) = \min(NLFrontNo(R_t))\}$;
$q = \arg\max\ (d_{min}(Q))$;
$P = P \cup q$, $R = R \backslash q$;
end
return P;

$O\left(MN\right)$, respectively. In the environment selection phase, the time complexity of computing the distance metric is $O\left(MN^2\right)$, and the complexity of filtering out N surviving solutions is $O\left(N\right)$. In addition, NLD-Based-Sort is required in both the mating selection and environment selection phases, which has a time complexity of $O\left(MN^2\right)$, so the total time complexity of each generation of NLDEA is $O\left(MN^2\right)$, which is comparable to some algorithms dealing with irregular PF problems.

RESULTS

This section focuses on the experimental comparison of a set of benchmark test problems and real-world problems.

Experimental Design

Algorithms in Comparison

In this study, the authors compared the performance of NLDEA with some state-of-the-art representative MaOEAs, including NSGA-III (Deb & Jain, 2013), θ-DEA (Yuan et al., 2015), MOEA/DD (K. Li et al., 2014), onebyoneEA (Y. Liu et al., 2017), NSGA-II/SDR (Tian et al., 2018), RVEA (R. Cheng et al., 2016), and MultiGPO (S. Zhu et al., 2021). In MOEA/DD, the neighborhood selection probability is $\delta = 0.9$, and the neighborhood size T and the maximum number of solutions each offspring replacement n_r are $\lceil 0.1N \rceil$ and $\lceil 0.01N \rceil$, respectively. According to (Yuan et al., 2015), θ-DEA in PBI uses $\theta = 5$. In onebyoneEA, the parameters k and R are set to $k = 0.1N$, and $R = 1$, respectively, following the guidelines in (Y. Liu et al., 2017). In MultiGPO, the parameters $\varphi = M$. In our method, NLDEA is $\alpha = 1.1$. The time complexity of MOEA/DD, NSGA-II/SDR, and RVEA, each generation is $O\left(MN^2\right)$, the time complexity of NSGA-III is max $\left\{O\left(Nlog^{M-2}N\right), O\left(MN^2\right)\right\}$, and the MultiGPO time complexity is $O\left(M^2N^2\right)$.

Test Problem

In this study, the authors used scalable test problems from two test suites, i.e., WFG1-WFG9 test problems with different sizes of objectives (Huband et al., 2006) and MaF1-MaF15 with complex PFs (R. Cheng et al., 2017). For each problem, they considered the number of objectives M varying between 5 and 20, i.e., M in {5, 10, 15, 20}. These test problems contained different problem properties, such as concavity, convexity, linearity, simplicity, and disconnectedness, and for each test instance, each algorithm was executed 20 times independently.

Parameter Settings

In all algorithms, SBX and PM were used to generate progeny. The crossover probability p_c was set to 1, the mutation probability p_m was set to $1/n$, and their distribution indices were set to 20. In principle, the population size N of NLDEA can be specified arbitrarily. However, NSGA-III, MOEA/DD, and θ-DEA require N to be the same as the number of relevant reference vectors, which cannot be set arbitrarily, because the number of reference lines generated by the Das and Dennis' method is the binomial coefficient (Das & Dennis, 1998). For a fair comparison, the overall population size of all algorithms was set to be the same for the same M, as can be seen in Table 1, and the maximum generation ($G_{\max}$) was used as the termination criterion for all algorithms, with the WFG problems set to 200 and the MAF problems set to 500.

Performance Metrics

The authors used IGD (Bosman & Thierens, 2003) as the performance metric. Note that all experiments in this study are executed on the open-source PlatEMO platform (Tian et al., 2017a, 2017b). IGD: Let $\mathcal{P}^*$ be a set of points uniformly distributed along PF. $\mathcal{P}$ denotes the final result obtained by the algorithm. Then the IGD value of $\mathcal{P}$ is defined as:

$$IGD\left(\mathcal{P}, \mathcal{P}^*\right) = \frac{\sum_{p\in\mathcal{P}^*} d\left(p, \mathcal{P}\right)}{\left|\mathcal{P}^*\right|} \tag{14}$$

where $d\left(p, \mathcal{P}\right)$ is the minimum Euclidean distance from p to $\mathcal{P}$ and $\left|\mathcal{P}^*\right|$ is the size of the set $\mathcal{P}^*$. The smaller the value of IGD, the better the performance of the algorithm is, of course. In these

Table 1. Main PF shapes of 24 test problems

Problem	M	n	G_{max}	PF Shapes
WFG test problems				
WFG1	5,10,15,20	$M-1+10$	200	Mixed, Biased, Scaled
WFG2				Convex, Disconnected, Multi-modal, Non-separable, Scaled Linear
WFG3				Degenerate, Non-separable, Scaled
WFG4				Concave, Multi-modal, Scaled
WFG5				Concave, Deceptive, Scaled
WFG6				Concave, Non-separable, Scaled
WFG7				Concave, Biased, Scaled
WFG8				Concave, Biased, Non-separable, Scaled
WFG9				Concave, Biased, Multi-modal, Deceptive, Non-separable, Scaled
MAF test problems				
MAF1	5,10,15,20	$M-1+10$	500	Linear
MAF2				Concave
MAF3				Convex, Multi-modal
MAF4				Concave, Multi-modal, Badly-scaled
MAF5				Convex, Biased, Badly-scaled
MAF6				Concave, Degenerate
MAF7		$M-1+20$		Mixed, Disconnected, Multi-modal
MAF8		2		Linear, Degenerate
MAF9				Linear, Degenerate
MAF10		$M-1+10$		Mixed, Biased
MAF11				Convex, Disconnected, Non-separable
MAF12				Concave, Non-separable, Biased Deceptive
MAF13		5		Concave, Unimodal, Non-separable, Degenerate, Complex Pareto set
MAF14		$20 \times M$		Linear, Partially separable, Large scale
MAF15				Convex, Partially separable, Large scale

experiments, about 5000 points were uniformly distributed over the PF generated by Das and Dennis (Das & Dennis, 1998) methods to form $\mathcal{P}^*$.

Statistical tests: to draw statistically reliable conclusions, the Wilcoxon rank sum test at the 0.05 significance level was used to analyze the differences between NLDEA and the other seven algorithms. The symbols "$+$", "$\approx$" and "$-$" indicate that the results of the other algorithms are significantly better than, similar, or worse than NLDEA, respectively. For clarity, the authors also used a bolded font to highlight the best algorithm for each instance.

Effect of NLD

The effectiveness of NLD was verified by embedding it into NSGA-III. In NSGA-III, convergence relies only on the traditional Pareto dominance, but with a limited budget, this is not sufficient to solve MaOP with a large number of objectives. For example, with a large number of objectives, NSGA-III usually requires a large number of iterations (e.g., $G_{\max} > 1000$) to obtain the desired results. Therefore, the authors replaced the Pareto advantage of NSGA-III with their proposed NLD scheme to enhance the selection pressure to determine the solution without losing diversity, which yields the new algorithm NSGA-III_NLD. Specifically, NSGA-III_NLD uses an improved environment selection step, as Algorithm 4 shows. In addition, the authors embed ($M-1$)-GPD and α-dominance into NSGA-III to obtain NSGA-III_(M-1)-GPD and NSGA-III_AD, respectively. In this section, they tested the WFG problems and the MAF problem. The decision variables and the number of iterations are shown in Table 1. S. Zhu et al. (2021) reported that in NSGA-III_(M-1)-GPD, the value of parameter φ was set to M, whereas in NSGA-III_AD, its value was set to $\alpha = \frac{1}{3}$ according to Ikeda et al. (2001), and in NSGA-III_NLD, the value of α was set to 1.1.

Table 2 and Table 3 report the IGD results of NSGA-III_Pareto, NSGA-III_AD, NSGA-III_(M-1)-GPD, NSGAII/SDR, and NSGA-III_NLD on the MAF problems and WFG problems, respectively. The best results for each instance are shown in bold font.

From the Table 2 it can be seen that NSGA-III_NLD outperforms or is similar to NSGA-III_Pareto, NSGA-III_AD, NSGA-III_(M-1)-GPD, and NSGAII/SDR in 57, 50, 47, and 32 out of 60 comparisons on the MAF problems, respectively.

Algorithm 4. Environmental selection of NSGA-III_NLD

Input: population size N, combined population R, the quadratic term coefficient λ, reference points Z
Output: survive population S
$S = \varnothing$, $i = 0$;
$(F_1, F_2, \ldots)$ =NLD-based-Sort(R, λ);
while $S < N$ **do**
$i = i + 1$;
$S = S \cup F_i$;
end
$S = S \setminus F_i$ and $S_L = F_i$ (S_L is the candidate solution set of the last front to be selected);
if $\lvert S\rvert + \lvert S_L\rvert = N$ **then**
$S = S \cup S_L$;
else
$\left(\tilde{S};\widetilde{S_L}\right) =$ Normalize(S, S_L)
d = Compute orthogonal distance to Z_i for each i;
ρ = Compute niche count of Z_i based on d for each i;
$S = S \cup$ Niching ($\widetilde{S_L}$, $N - \lvert S\rvert$, ρ, d);
end
return S

Table 2. IGD Results of NSGA-III_Pareto, NSGA-III_AD, NSGA-III_(M-1)-GPD, NSGAII/SDR, and NSGAIII_NLD on MAF1-MAF15 with $G_{\max} = 500$

Problem	*M*	NSGA-III_Pareto	NSGA-III_AD	NSGA-III_(M-1)-GPD	NSGAII/SDR	NSGA-III_NLD
MaF1	5	1.8541e-1(1.42e-2) ≈	1.7259e-1 (7.96e-3) +	1.8446e-1 (1.11e-2) ≈	**1.0419e-1 (1.09e-3) +**	1.8608e-1 (1.25e-2)
	10	2.7865e-1 (4.46e-3) ≈	2.7509e-1 (5.30e-3) ≈	2.7841e-1 (5.31e-3) ~	**2.1532e-l (1.29e-3) +**	2.7799e-1 (4.58e-3)
	15	3.1799e-1 (9 90e-3) ≈	3.3452e-1 (1.15e-2) -	3.1394e-1 (4.74e-3) +	**2.8222e-l (4.2le-3) +**	3.2038e-1 (8.19e-3)
	20	4.4485e-1 (1 .24e-2) ≈	4.3989e-1 (9.19e-3) ≈	4.2874e-1 (6.84e-3) +	**3.4929e-l (2.60e-3) +**	4. 4641e-1 (8.10e-3)
MaF2	5	1.1192e-1 (2.26e-3) ≈	1.5943e-1(3.25e-3) -	1.1051e-1 (2.09e-3) ≈	**9.4872e-2 (1.48e-3) +**	1.1125e-1 (2.81e-3)
	10	**2.0307e-1 (1.41e-2) ≈**	5.1518e-1(3.34e-3) -	2.0941e-1 (2.32e-2) ≈	2.2164e-1 (1.36e-2) -	2.1060e-1 (2.29e-2)
	15	2.0924e-1 (6.76e-3) ≈	5.8619e-1(1.15e-2) -	**2.0566e-1 (8.72e-3) ≈**	3.5154e-1 (3.63e-2) -	2.0615e-1 (6.59e-3)
	20	3.2043e-1 (4.24e-2) ≈	6.1271e-1 (1.45e-2) -	**3.0093e-1 (4.07e-2) ≈**	3.6486e-1 (1.72e-2) -	3.1719e-1 (4.84e-2)
MaF3	5	7.7889e-2 (7.04e-3) -	2.0269e-1(5.31e-3) -	**7.2452e-2 (1.74e-2) ≈**	1.4715e-1 (5.86e-3) -	7.6077e-2 (2.11e-2)
	10	3.0511e+3 (885e+3) -	1.6730e-1(1.33e-3) -	**9.9092e-2 (5.67e-3) ≈**	1.5024e-1 (3.30e-3) -	1.0105e-1 (8.86e-3)
	15	3.3975e+2 (1.06e+3) -	1.5182e-1(4.23e-4) -	**1.0200e-1 (5.35e-3) ≈**	1.4097e-1 (1.37e-3) -	1.0541e-1 (7.60e-3)
	20	4.1892e+4 (6.06e+4) -	2.0760e-1(5.64e-4) -	7.3490e+0 (2.60e+1) ≈	1.9857e-1 (9.73e-4) -	**1.6689e-1 (1.30e-2)**
MaF4	5	2.7846e+0 (1.04e+0) ≈	1.1841e+1 (2.71e-1) -	2.5378e+0 (6.40e-1) ≈	**2.2393e+0 (959e-2) +**	2.3815e+0 (9.33e-2)
	10	9.6209e+1 (6.91e+0) ≈	5.0161e+2 (1.09e+1) -	**8.2455e+1 (7.32e+0) +**	1.7767e+2 (2.86e+1) -	9.4943e+1 (9.26e+0)
	15	3.8586e+3 (2.94e+2) ≈	1.7770e+4 (2.77e+2) -	**3.1531e+3 (5.01e+2) +**	7.2591e+3 (1.61e+3) -	3.8737e+3 (2.75e+2)
	20	**1.3542e+5 (1 81e+4) ≈**	6.5139e+5 (2.06e+4) -	1.4269e+5 (5 00e+4) ≈	3.1861e+5 (6.27e+4) -	1.5228e+5 (2.71e+4)
MaF5	5	1.9699e+0 (2.68e-3) ≈	1.4635e+1 (1.04e-5) -	**1.9465e+0 (1.25e-2) +**	1.2413e+1 (2.71e+0) -	2.5317e+0 (1.38e+0)
	10	**7.8111e+1 (8.29e-1) +**	3.0631e+2 (4.47e-5) -	2.8552e+2 (1.06e+1) -	3.0600e+2 (1.42e+0) -	8.7431e+1 (3.54e+1)
	15	2.4563e+3 (9.91e+1) -	7.3261e+3 (4.94e-5) -	7.3117e+3 (8.27e+0) -	7.3261e+3 (2.42e-2) -	**2.0617e+3 (3.30e+2)**
	20	7.4568e+4 (1.71e+4) -	1.7095e+5 (1.15e-3) -	1.7029e+5 (2.96e+3) -	1.7095e+5 (1.05e-1) -	**5. 8238e+4 (2.36e+4)**
MaF6	5	1.6847e-2 (5.17e-3) ≈	3 4242e-1 (2.74e-8) -	1.6295e-2(3.64e-3) ≈	**1.5610e-2 (1.33e-2) +**	1.7078e-2 (3.69e-3)
	10	7.1234e-1 (3.02e-1) -	3. 4182e-1 (2.69e-3) -	1.5410e-2 (9.05e-3) ≈	**1.1906e-2 (9.31e-3) ≈**	1.3112e-2 (6.76e-3)
	15	7.8246e-1 (2.80e-1) -	3. 4219e-1 (7.08e-4) -	1.6803e-1 (2.31e-1) -	1.9506e-2 (6.67e-2) -	**1.8454e-2 (6.72e-3)**
	20	6.8526e+0 (8.59e+0) -	3. 4059e-1 (8.18e-3) -	1.7167e-1 (2.11e-1) ≈	9.8692e-2 (1.34e-1) ≈	**4.3674e-2 (4.06e-2)**
MaF7	5	**2.8099e-1 (7.29e-3) ≈**	1.0817e+0 (1.45e-2) -	2.8285e-1 (6.47e-3) ≈	3.2404e-1 (2.36e-2) -	2.8102e-1 (7.70e-3)
	10	1.0769e+0 (7.21e-2) -	5.6673e+0 (1.92e-2) -	**9.5023e-1 (5.30e-2) ≈**	1.5573e+0 (2.50e-1) -	9.6917e-1 (4.78e-2)
	15	4.1741e+0 (4.94e-1) -	8.6308e+0 (3.84e-2) -	3.1417e+0 (4.32e-1) ≈	4.1605e+0 (4.00e-1) -	**3.0175e+0 (3.51e-1)**
	20	8.3123e+0 (8.72e-1) ≈	1.2899e+1 (7.22e-2) -	7.6548e+0 (8.35e-1) +	**7.3432e+0 (5.59e-1) +**	8.3870e+0 (1.02e+0)
MaF8	5	1.5849e-1 (9.67e-3) ≈	1.8791e-1 (1.39e-2) -	1.5897e-1 (9.29e-3) ≈	**9.7832e-2 (4.56e-3) +**	1.5557e-1 (8.59e-3)
	10	3.1987e-1 (6.91e-2) ≈	6.9761e-1 (1.51e-2) -	3.2387e-1 (7.69e-2) ≈	**1.4337e-1 (8.80e-3) +**	3.1273e-1 (7.54e-2)
	15	4.0866e-1 (1.04e-1) ≈	1.1247e+0 (1.59e-2) -	3.9803e-1 (7.76e-2) ≈	**2.0245e-1 (1.77e-2) +**	4.3381e-1 (1.05e-1)
	20	4.5365e-1 (5.44e-2) ≈	1.5308e+0 (5.13e-2) -	4.4511e-1 (7.12e-2) ≈	**2.6280e-1 (3.28e-2) +**	4.4786e-1 (6.09e-2)
MaF9	5	4.1393e-1 (1.75e-1) -	**1.3148e-1 (1.43e-2) +**	4.2345e-1 (2.06e-1) -	1.3419e-1 (5.63e-3) +	1.4647e-1 (1.47e-2)
	10	5.3683e-1 (1.01e-1) -	2.3461e-1 (1.81e-2) +	3.9639e-1 (9.46e-2) -	**1.6966e-1 (6.51e-3) +**	2.7520e-1 (5.66e-2)
	15	3.7716e-1 (6.54e-2) ≈	2.6110e+0 (4.70e+0) ≈	4.1481e-1 (1.06e-1) -	**1.9102e-1 (4.84e-3) +**	3.6292e-1 (6.96e-2)
	20	1.3550e+1 (8.76e+0) -	1.9845e+0 (4.67e+0) +	3.6973e+0 (6.58e+0) +	**2.3142e-1 (6.03e-3) +**	5.0498e+0 (8.81e+0)
MaF10	5	**3.7112e-1 (5.84e-3) ≈**	1.4782e+0 (2.43e-2) -	3.7323e-1 (6.09e-3) ≈	6.8762e-1 (I.13e-1) -	3.7363e-1 (8.86e-3)
	10	**1.0268e+0 (5.95e-2) ≈**	2.1280e+0 (8.42e-3) -	1.4702e+0 (5.03e-1) ≈	1.7852e+0 (1.20e-1) -	1.1808e+0 (3.36e-1)
	15	**1.5572e+0 (1.02e-1) +**	2.6953e+0 (3.68e-3) -	1.9477e+0 (3. 43e-1) -	2.4500e+0 (3.92e-2) -	1.6482e+0 (1. 91e-1)
	20	4.3742e+0 (4.69e-1) ≈	5.4028e+0 (3.38e-3) -	**4.1938e+0 (1.19e-1) +**	5.1637e+0 (3.42e-2) -	4.6181e+0 (3.40e-1)
MaF11	5	3.8900e-1 (1.37e-3) -	1.4645e+0 (1.29e-2) -	3.8590e-1 (2.18e-3) -	5.1186e-1 (4.52e-2) -	**3.8349e-1 (2.27e-3)**
	10	**1.2755e+0 (8.19e-2) ≈**	2.2860e+0 (7.22e-3) -	1.3978e+0 (1.50e-1) ≈	1.5711e+0 (1.01e-1) -	1.3212e+0 (1.12e-1)
	15	**1.5354e+0 (5.63e-2) +**	2.8711e+0 (3.32e-3) -	1.8074e+0 (1.31e-1) -	2.3591e+0 (6.86e-2) -	1.5911e+0 (9.13e-2)
	20	4.0010e+0 (1.43e-1) ≈	5.6734e+0 (4.49e-3) -	4.3897e+0 (2.29e1) -	5.1872e+0 (6.81e-2) -	**3.9338e+0 (1.95e-1)**

continued on following page

Table 2. Continued

Problem	M	NSGA-III_Pareto	NSGA-III_AD	NSGA-III_(M-1)-GPD	NSGAII/SDR	NSGA-III_NLD
MaF12	5	9.3559e-1 (2.08e-3) ≈	5.8411e+0 (2.75e-1) -	9.3471e-1 (2.69e-3) ≈	9.7513e-1 (8.10e-3) -	**9.3435e-1 (3.12e-3)**
	10	4.4051e+0 (2.24e-2) ≈	1.1923e+1 (7.79e-2) -	4.4013e+0 (3.27e-2) ≈	**4.3188e+0 (2.95e-2) +**	4.4034e+0 (5.59e-2)
	15	7.9275e+0 (1.39e-1) ≈	1.7389e+1 (1.62e-1) -	7.8143e+0 (2.21e-1) +	**7.6645e+0 (1.50e-1) +**	7.9608e+0 (1.58e-1)
	20	1.3804e+1 (8.48e-1) ≈	2.1245e+1 (2.39e+0) -	1.4305e+1 (8.07e 1) -	**1.2822e+1 (1.52e+0) +**	1.3786e+1 (8.05e-1)
MaF13	5	2.0715e-1 (2.31e-2) -	4. 4908e-1 (6.59e-2) -	1.3376e-1 (1.35e-2) ≈	1.3911e-+ (1.05e-2) -	**1.2754-1 (8.87e-3)**
	10	2.4097e-1 (2.08e-2) -	1.5437e+0 (1.07e-2) -	1.9624e-1 (2.15e-2) ≈	**1.6769e-1 (1.21e-2) +**	1.8650e-1 (1.56e-2)
	15	2.9155e-1 (3.70e-2) -	1.9225e+0 (2.46e-2) -	2.5533e-1 (4. 36e-2) -	**1.7187e-1 (1.14e-2) +**	2.1671e-1 (2.38e-2)
	20	3.0198e-1 (2.46e-2) -	2.2234e+0 (3.22e-2) -	2.9012e-1 (2.51e-2) -	**1.7820e-1 (1.16e-2) +**	2.6855e-1 (2.41e-2)
MaF14	5	1.7678e+0 (1.19e+0) -	9.1182e-1 (1.07e-1)-	**5.1638e-1 (7.72e-2) +**	5.4829e-1 (1.21e-1) +	7.8448e-1 (2.27e-1)
	10	1.0779e+1 (6.47e+0) -	1.0250e+0 (2.01e-4) ≈	**9.1903e-1 (136e-1) +**	1.1130e+0 (1.31e-1) -	1.0148e+0 (3.52e-2)
	15	3.6909e+0 (4.28e+0) -	**9.9434e-1 (1.17e-1) +**	1.2934e+0 (3.61e-1) ≈	1.2677e+0 (2.28e-1) ≈	1.5842e+0 (1.20e+0)
	20	4.1403e+0 (4.89e+0) -	1.0963e+0 (8.15e-4) +	**9.6524e-1 (2.05e-1) +**	1.0682e+0 (1.72e-2) +	1.1630e+0 (1.28e-1)
MaF15	5	1.1652e+0 (1. 34e-1) -	**6.3751e-1 (4. 84e-2) +**	1.0027e+0 (1 64e-1) ≈	7.9559e-1 (2.66e-2) +	9.3407e-1 (5.29e 2)
	10	2.1917e+0 (7.87e-1) ≈	**1.0328e+0 (3.62e-2) +**	1.5166e+0 (2.85e-1) +	1.0812e+0 (2.39e-2) +	1.9786e+0 (3. 65e-1)
	15	1.0687e+1 (4.30e+0) -	**1.1773e+0 (3.10e-2) +**	3.9332e+0 (1.12e+0) -	1.2629e+0 (3.69e-2) +	2.6388e+0 (3.17e-1)
	20	1.8457e+1 (6.80e+0) -	**1.2596e+0 (2.88e-2) +**	5.5056e+0 (3.60e+0) ≈	1.4278e+0 (4.79e-2) +	5.3470e+0 (8.65e-1)
+/ − / ≈		3/26/31	10/46/4	13/15/32	28/29/3	

From Table 3 it can be seen that NSGA-III_NLD outperforms or is similar to NSGA-III_Pareto, NSGA-III_AD, NSGA-III_(*M*-1)-GPD, and NSGAII/SDR in 35, 35, 34, and 25 out of 36 comparisons on the WFG problems, respectively. These results confirm the effectiveness of the NLD selection criterion. It can also be easily embedded into other existing MaOEAs, thus balancing convergence and diversity well.

Performance Comparison to Other MaOEAs

Table 4 displays the results of the NLD-based many-objective evolutionary algorithm NLDEA on WFG benchmark functions. It can be seen that among the 36 comparisons, NLDEA obtained the best or equal to the best results 14 times, θ-DEA obtained the best results 11 times; while NSGA-III, RVEA, NSGA-II/SDR, onebyoneEA, MOEA/DD, and MultiGPO performed on 2, 3, 1, 0, 2, and 3 times, respectively.

Table 5 reports the IGD results for all methods on MaF1-MaF15. From these comparisons, it can be seen that in most cases, NLD achieves better IGD performance than other MaOEAs overall. For IGD, NLDEA outperforms NSGA-III, θ-DEA, RVEA, NSGAII/SDR, onebyoneEA, MOEA/DD, and MultiGPO in 49, 51, 44, 48, 51, 52, and 31 out of 60 cases, respectively. However, NLDEA is inferior to MultiGPO on the MaF8 - MaF11 problems. This is because MultiGPO applies *M* symmetrical cases of (M -1)-GPD, where each enhances the selection pressure of "*M*-1" objectives by expanding the dominance area of solutions, while remaining unchanged for the one objective left out of that process. However, the time complexity ($O\left(M^2N^2\right)$) is larger compared to NLDEA ($O\left(MN^2\right)$). For the reader's intuitive understanding, Figure 3 shows the parallel coordinate plots of the objective values of NLDEA for the ten-objective MaF1-MAF7 and WFG7-WFG9 problems, from which it can be observed that NLDEA can obtain widely distributed solutions in most cases.

Sensitivity Analysis of Impact Factor α

Here, the authors try to further investigate the influence of the parameter α on the performance of NLD. Figure 4 reports the effect of setting different values of α in NLDEA on the IGD values of

Table 3. IGD results of NSGA-III_Pareto, NSGA-III_AD, NSGA-III_(M-1)-GPD, NSGAII/SDR and NSGA-III_NLD on WFG1-WFG9 with $G_{\max} = 200$

Problem	*M*	NSGA-III_Pareto	NSGA-III_AD	NSGA-III_(M-1)-GPD	NSGAII/SDR	NSGA-III_NLD
WFG1	5	6.4633e-1 (6.51e-2) ≈	1.5451e+0 (4.02e-2) -	6.02220-1 (6.03e-2) +	**5.8801e-1 (5.81e-2) +**	6.4727e-1 (6.78e-2)
	10	1.5279e+0 (8.16e-2) ≈	2.1502e+0 (1.05e-2) -	1.5328e+0 (1.83e-1) ≈	1.5830e+0 (1.08e-1) ≈	**1.5173e+0 (1.13e-1)**
	15	**2.0926e+0 (7.78e-2) ≈**	2.6963e+0 (5.29e-3) -	2.1215e+0 (1.43e-1) ≈	2.3540e+0 (5.88e-2) -	2.1379e+0 (9.39e-2)
	20	**4.4720e+0 (1.86e-1) ≈**	5.4035e+0 (4.10e-3) -	4.7158e+0 (1.76e-1) -	5.1091e+0 (5.45e-2) -	4.5318e+0 (1.42e-1)
WFG2	5	3.8788e-1 (3.33e-3) -	1.4612e+0 (134e-2) -	3.8494e-1 (3.56e-3) ≈	4.8984e-1 (3.70e-2) -	**3.8216e-1 (4.27e-3)**
	10	**1.2586e+0 (1.87e-1) +**	2.2907e+0 (6.79e-3) -	1.3833e+0 (1.57e-1) ≈	1.6417e+0 (1.22e-1) -	1 3621e+0 (1.64e-1)
	15	1.5433e+0 (7.53e-2) ≈	2.8692e+0 (4.23e-3) -	1.7261e+0 (1.05e-1) -	2.3390e+0 (9.14e-2) -	**1.5419e+0 (6.67e-2)**
	20	**3.8676e+0 (1.48e-1) ≈**	5.6782e+0 (5.37e-3) -	4.1100e+0 (3.08e-1) -	5.1894e+0 (1.11e-1) -	3.9118e+0 (1.71e-1)
WFG3	5	5.0044e-1 (5.69e-2) ≈	**2.7274e-1 (2.69e-1) +**	4.9622e-1 (4.83e-2) ≈	3.5982e-1 (4.03e-2) +	4.9503e-1 (6.26e-2)
	10	**1.1143e+0 (2.22e-1) ≈**	5.1145e+0 (9.72e-1) -	1.1491e+0 (2.84e-1) ≈	1.4274e+0 (4.02e-1) -	1.1282e+0 (2.21e-1)
	15	2.3812e+0 (3.63e-1) ≈	1.0239e+1 (6.04e+0) -	**1.9188e+0 (3.92e-1) ≈**	4.3442e+0 (1.89e+0) -	2.1418e+0 (4.36e-1)
	20	5.7969e+0 (2.14e+0) ≈	8.4328e+0 (2.57e+0) -	**5.5872e+0 (2.61e+0) ≈**	7.7971e+0 (2.02e+0) -	6.1992e+0 (2.34e+0)
WFG4	5	9.6477e-1 (1.78e-3) ≈	6.0956e+0 (2.74e-4) -	9.6619e-1 (3.45e-3) -	9.9752e-1 (9.44e-3) -	**9.6427e-1 (2.50e-3)**
	10	4.5272e+0 (2.46e-2) ≈	1.2005e+1 (4.69e-4) -	4.5230e+0 (7.61e-2) +	**4.3470e+0 (3.63e-2) +**	4.5237e+0 (1.56e-2)
	15	8.1718e+0 (1.04e-1) ≈	1.7616e+1 (2.43e-2) -	8.2055e+0 (9.49e-2) ≈	**8.0406e+0 (2.01e-1) ≈**	8.1410e+0 (1.07e-1)
	20	**1.4511e+1 (1.26e+0) ≈**	1.7552e+1 (1.33e+0) -	1.4766e+1 (9.75e-1) ≈	1.6210e+1 (3.13e+0) ≈	1.4514e+1 (1.54e+0)
WFG5	5	9.4864e-1 (3.65e-3) ≈	6.0360e+0 (4.65e-4) -	**9.4815e-1 (3.29e-3) ≈**	9.8991e-1 (1.03e-2) -	9.4858e-1 (3.56e-3)
	10	4.4688e+0 (1.46e-2) ≈	1.1923e+1 (7.45e-4) -	4.4848e+0 (2.17e-2) -	**4.3984e+0 (6.25e-2) +**	4.4621e+0 (1.80e-2)
	15	7.9944e+0 (2.64e-1) ≈	1. 7518e+1 (152e-2) -	7.9349e+0 (3.13e-1) ≈	8.1242e+0 (2.23e-1) -	**7.8618e+0 (3.28e-1)**
	20	1.2696e+1 (6.09e-1) ≈	2.2915e+1 (3.20e-1) -	1.2939e+1 (7.30e-1) ≈	**1.2644+1 (1.52e+0) +**	1.2980e+1 (8.51e-1)
WFG6	5	9.6683e-1 (4.05e-3) ≈	6.0210e+0 (1.78e-2) -	9.6638e-1 (4.14e-3) ≈	1.0030e+0 (1.25e-2) -	**9.6547e-1 (2.86e-3)**
	10	4.5782e+0 (2.03e-2) ≈	1. 1893e+1 (2.02e-2) -	4.6122e+0 (2.22e-2) -	**4.5428e+0 (5.05e-2) +**	4.5814e+0 (1.87e-2)
	15	8.1303e+0 (2.96e-1) ≈	1.7512e+1 (3.97e-2) -	8.1534e+0 (4.04e-1) ≈	8.2207e+0 (2.22e-1) -	**8.1283e+0 (4.75e-1)**
	20	1.3441e+1 (1.09e+0) ≈	2.3013e+1 (2.42e-2) -	1.3844e+1 (7.27e-1) -	1.8165e+1 (1.96e+0) -	**1.3158e+1 (8.42e-1)**
WFG7	5	9.6464e-1 (2.62e-3) ≈	6.0961e+0 (1.03e-4) -	**9.6442e-1 (1.84e-3) ≈**	9.9472e-1 (1.11e-2) -	9.6485e-1 (1.67e-3)
	10	4.5330e+0 (6.3le-2) ≈	1.2006e+1 (5.58e-4) -	4.5599e+0 (6.58e-2) ≈	**4.4166e+0 (5.34e-2) +**	4.5335e+0 (4.31e-2)
	15	8.1228e+0 (9.62e-2) ≈	1.7600e+1 (2.73e-2) -	8.1285e+0 (1.05e-1) ≈	9.0640e+0 (1.29e+0) -	**8.0996e+0 (8.31e-2)**
	20	1.6287e+1 (9.87e-1) ≈	2.0271e+1 (2.47e+0) -	1.6543e+1 (6.84e-1) ≈	**1.3992e+1 (9.30e-1) +**	1.6378e+1 (9.74e-1)
WFG8	5	1.0012e+0 (7.98e-3) ≈	4.4721e+0 (7.07e-2) -	1.0029e+0 (4.41e3) ≈	1.0498e+0 (7.88e-3) -	**1.0008e+0 (7.27e-3)**
	10	4.5863e+0 (3.05e-1) ≈	1.1453e+1 (3.48e-1) -	4.5752e+0 (3.12e-1) ≈	4.5036e+0 (6.60e-2) ≈	**4.4975e+0 (2.73e-1)**
	15	8.2927e+0 (5.290-1) ≈	1.2858e+1 (2.48e+0) -	8.3105e+0 (5.16e-1) ≈	8.8589e+0 (1.25e+0) ≈	**8.2228e+0 (5.00e-1)**
	20	1.8290e+1 (1.93e+0) ≈	1.8134e+1 (1.92e+0) ≈	1.8251e+1 (1.71e+0) ≈	**1.4130e+1 (1.80e+0) +**	1.8864e+1 (1.71e+0)
WFG9	5	9.3349e-1 (5.23e-3) ≈	5.7416e+0 (2.74e-1) -	**9.3062e-1 (5.05e-3) ≈**	9.7647e-1 (1.0le-2) -	9.3197e-1 (4.90e-3)
	10	4.2950e+0 (5.73e-2) ≈	1. 1672e+1 (8.90e-2) -	4.3052e+0 (5.90e-2) ≈	**4.2827e+0 (4.69e-2) ≈**	4 3238e+0 (6.71e-2)
	15	7.9971e+0 (1.52e-1) ≈	1.6505e+1 (5.27e-1) -	7.9844e+0 (1.53e-1) ≈	**7.6602e+0 (132e-1) +**	7.9799e+0 (2.10e-1)
	20	1.4364e+1 (7.20e-1) ≈	1.8314e+1 (1.63e+0) -	1.4234e+1 (7.03e-1) ≈	**1.3285e+1 (7.95e-1) +**	1 4456e+1 (7.08e-1)
+/ − / ≈		1/1/34	1/34/1	2/7/27	11/19/6	

WFG problems with different number of objectives. For $\alpha = \left[0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\right]$, while $M \in \left\{5, 10, 15, 20\right\}$. From Figure 4 it can be seen that the hyper-parameter α does not have a significant effect on the IGD value while it is not equal to 0. In other words, the validity of the dynamic change of the quadratic term coefficient λ is proved and the effect of the hyperparameter α on the algorithm NLDEA can be reduced.

Table 4. IGD results of NSGA-III, θ-DEA, RVEA, NSGAII/SDR, onebyoneEA, MOEA/DD, MultiGPO and NLDEA on WFG1-WFG9 with $G_{\max} = 200$

Problem	M	NSGA-III	θ-DEA	RVEA	NSGAII/SDR	onebyoneEA	MOEA/DD	MultiGPO	NLDEA
WFG1	5	6.4633e-1 (6.51e-2) +	**4.7995e-1 (3.06e-2) +**	6.0303e-1 (1.14e-1) +	5.8801e-1 (5.81e-2) +	7.6730e-1 (7.94e-2) -	7.4786e-1 (9.39e-2) ≈	5.1364e-1 (6.82e-2) +	7.1386e-1 (3.39e-2)
	10	1.5279e+0 (8.16e-2) ≈	**1.1255e+0 (6.62e-2) +**	1.1293e+0 (7.62e-2) +	1.5830e+0 (1.08e-1) -	1.6125e+0 (8.21e-2) -	1.4456e+0 (8.30e-2) ≈	1.2915e+0 (1.35e-1) +	1.4867e+0 (1.21e-1)
	15	2.0926e+0 (7.78e-2) ≈	**1.7327e+0 (5.15-2) +**	1.8833e+0 (6.06e-2) +	2.3540e+0 (5.88e-2) -	2.2587e+0 (1.21e-1) -	2.2632e+0 (1.31e-1) -	1.8935e+0 (1.25e-1) +	2.1428e+0 (9.14e-2)
	20	4.4720e+0 (1.86e-1) ≈	**3.7718e+0 (2.54e-1) +**	4.2362e+0 (1.83e-1) +	5.1091e+0 (5.45e-2) -	4.6511e+0 (2.50e-1) ≈	4.8843e+0 (9.95e-2) -	4.1742e+0 (3.55e-1) +	4.5417e+0 (1.59e-1)
WFG2	5	3.8788e-1 (3.33e-3) +	**3.8661e-1 (4.81e-3) +**	3.9699e-1 (1.97e-2) +	4.8984e-1 (3.70e-2) +	6.6356e-1 (5.67e-2) -	4.7721e-1 (1.30e-2) +	3.9599e-1 (2.04e-2) +	5.5790e-1 (7.53e-2)
	10	1.2586e+0 (1.87e-1) +	**1.1190e+0 (4.74e-2) +**	1.1358e+0 (4.76e-2) +	1.6417e+0 (1.22e-1) ≈	1.7412e+0 (7.10e-2) -	1.3254e+0 (4.09e-2) +	1.4523e+0 (7.99e-2) +	1.6114e+0 (7.51e-2)
	15	**1.5433e+0 (7.53e-2) +**	3.8713e+0 (1.36e+0) -	1.6699e+0 (1.05e-1) +	2.3390e+0 (9.14e-2) -	2.2015e+0 (9.21e-2) ≈	1.9466e+0 (5.51e-2) +	2.0763e+0 (1.00e-1) +	2.2492e+0 (1.16e-1)
	20	3.8676e+0 (1.48e-1) +	4.6667e+0 (5.33e-1) ≈	**3.4381e+0 (2.57e-1) +**	5.1894e+0 (1.11e-1) -	4.6553e+0 (2.22e-1) ≈	5.4781e+0 (6.42e-2) -	4.2857e+0 (3.22e-1) +	4.7074e+0 (2.64e-1)
WFG3	5	5.0044e-1 (5.69e-2) +	4.7065e-1 (6.62e-2) +	5.3267e-1 (8.75e-2) +	**3.5982e-1 (4.03e-2) +**	1.3942e+0 (8.48e-2)-	6.2437e-1 (4.74e-2) +	9.1689e-1 (1.05e-1) -	8.0520e-1 (8.16e-2)
	10	1.1143e+0 (2.22e-1) +	**1.0867e+0 (1.52e-1) +**	3.7229e+0 (8.89e-1) -	1.4274e+0 (4.02e-1) +	5.2702e+0 (6.21e-1) -	2.8005e+0 (8.38e-2) -	2.6471e+0 (5.78e-1) -	2.3585e+0 (2.36e-1)
	15	2.3812e+0 (3.63e-1) +	**2.2696e+0 (5.44e-1) +**	6.0206e+0 (1.17e+0) -	4.3442e+0 (1.89e+0) ≈	1.0421e+1 (2.18e+0) -	6.0341e+0 (3.17e-1) -	2.2854e+0 (5.57e-1) +	3.4676e+0 (7.33e-1)
	20	5.7969e+0 (2.14e+0) -	3.5612e+0 (5.00e-1) -	9.1077e+0 (1.37e+0) -	7.7971e+0 (2.02e+0) -	1.3064e+1 (2.50e+0) -	2.0362e+1 (5.13e-1) -	**1.8000e+0 (3.03e-1) +**	2.1486e+0 (5.30e-1)
WFG4	5	9.6477e-1 (1.78e-3) -	9.6473e-1 (2.60e-3) -	9.6131e-1 (1.31e-3) -	9.9752e-1 (9.44e-3) -	1.4499e+0 (1.19e-1) -	1.0519e+0 (2.94e-3) -	9.6339e-1 (9.84e-3) -	**9.4633e-1 (7.22-3)**
	10	4.5272e+0 (2.46e-2) -	4.5203e+0 (1.33e-2) -	4.4015e+0 (6.08e-2) -	4.3470e+0 (3.63e-2) -	5.8485e+0 (1.68e-1) -	6.2464e+0 (1.22e-1) -	4.0456e+0 (2.98e-2) -	**3.9985e+0 (1.95-2)**
	15	8.1718e+0 (1.04e-1) -	8.0956e+0 (8.32e-2) -	8.9085e+0 (3.62e-1) -	8.0406e+0 (2.01e-1) -	1.0785e+1 (3.10e-1) -	9.1835e+0 (2.62e-1) -	**7.3922e+0 (5.82e-2) ≈**	7.4356e+0 (8.91e-2)
	20	1.4511e+1 (1.26e+0) -	**1.1298e+1 (5.10e-2) +**	1.1622e+1 (3.98e-1) +	1.6210e+1 (3.13e+0) -	1.5331e+1 (4.95e-1) -	1.1473e+1 (5.81e-1) +	1.1787e+1 (2.65e-1) ≈	1.1959e+1 (3.81e-1)
WFG5	5	9.4864e-1 (3.65e-3) -	9.4837e-1 (3.60e-3) -	9.5179e-1 (1.37e-3) -	9.8991e-1 (1.03e-2) -	1.4138e+0 (9.68e-2) -	1.0355e+0 (4.08e-3) -	9.5057e-1 (6.61e-3) -	**9.4374e-1 (9.96e-3)**
	10	4.4688e+0 (1.46e-2) -	4.4605e+0 (1.88e-2) -	4.3635e+0 (5.23e-2) -	4.3984e+0 (6.25e-2) -	5.9759e+0 (1.97-1) -	6.3908e+0 (9.54e-2) -	4.0262e+0 (3.94e-2) -	**3.9967e+0 (3.12e-2)**
	15	7.9944e+0 (2.64e-1) -	7.6221e+0 (6.28e-2) -	8.4939e+0 (1.75e-1) -	8.1242e+0 (2.23e-1) -	1.1103e+1 (1.91e-1) -	9.3269e+0 (2.18e-1) -	7.1859e+0 (9.00e-2) -	**7.1289e+0 (6.14e-2)**
	20	1.2696e+1 (6.09e-1) -	**1.1172e+1 (3.66e-2) +**	1.3456e+1 (2.34e-1) -	1.2644e+1 (1.52e+0) -	1.5656e+1 (2.36e-1) -	1.8759e+1 (3.10e-1) -	1.1735e+1 (2.02e-1) -	1.1267e+1 (1.43e-1)
WFG6	5	9.6683e-1 (4.05e-3) ≈	9.6553e-1 (332e-3) +	**9.6479e-1 (2.60e-3) +**	1.0030e+0 (1.25e-2) -	1.7470e+0 (1.26e-1) -	1.0457e+0 (3.42e-3) -	1.0168e+0 (1.95e-2) -	9.7396e-1 (1 .08e-2)
	10	4.5782e+0 (2.03e-2) -	4.5771e+0 (1.53e-2) -	4.3895e+0 (7.60e-2) -	4.5428e+0 (5.05e-2) -	6.7786e+0 (1.94e-1) -	6.1654e+0 (1.33e-1) -	4.2239e+0 (7.45e-2) -	**4.1350e+0 (2.83e-2)**
	15	8.1303e+0 (2.96e-1) -	7.9489e+0 (1.83e-1) -	9.0687e+0 (3.41e-1) -	8.2207e+0 (2.22e-1) -	1.2588e+1 (4.94e-1) -	1.0638e+1 (3.54e-1) -	7.3610e+0 (2.26e-1) -	**7.0709e+0 (6.37e-2)**
	20	1.3441e+1 (1.09e+0) -	1.1461e+1 (2.67e-2) -	1.4254e+1 (1.00e+0) -	1.8165e+1 (1.96e+0) -	1.7624e+1 (5.98e-1) -	1.5810e+1 (3.41e+0) -	1.1259e+1 (2.14e-1) -	**1.0858e+1 (1.35e-1)**
WFG7	5	9.6464e-1 (2.62e-3) -	9.6439e-1 (2 28e-3) -	9.6464e-1 (1.66e-3) -	9.9472e-1 (1.11e-2) -	1.8132e+0 (1.35e-1) -	1.0600e+0 (4.21e-3) -	9.6261e-1 (8.07e-3) -	**9.5231e-1 (8.62e-3)**
	10	4.5330e+0 (6.31e-2) -	4.5560e+0 (2.34e-2) -	4.3168e+0 (5.88e-2) -	4.4166e+0 (5.34e-2) -	6.0225e+0 (2.10e-1) -	5.4779e+0 (3.26e-1) -	4.0450e+0 (2.23e-2) -	**3.9976e+0 (1.86-2)**
	15	8.1228e+0 (9.62e-2) -	8.2475e+0 (5.85e-2) -	8.0017e+0 (4.31e-1) -	9.0640e+0 (1.29e+0) -	9.7990e+0 (2.69e-1) -	8.2405e+0 (5.35e-1) -	7.2655e+0 (4.99e-2) -	**7.1992e+0 (4.89e-2)**
	20	1.6287e+1 (9.87e-1) -	1.1884e+1 (9.15e-2) -	1.1819e+1 (2.84e-1) -	1 3992e+1 (9.30e-1) -	1.3629e+1 (1.92e-1) -	**1.1464e+1 (1.57e-1) ≈**	1.1937e+1 (1.27e-1) -	1.1545e+1 (1.94e-1)

continued on following page

Table 4. Continued

Problem	M	NSGA-III	θ-DEA	RVEA	NSGAII/SDR	onebyoneEA	MOEA/DD	MultiGPO	NLDEA
WFG8	5	1.0012e+0 (7.98e-3) +	**9.9778e-1 (6.04e-3) +**	1.0057e+0 (2.66e-3) +	1.0498e+0 (7.88e-3) +	1.5819e+0 (1.05e-1) -	1.0652e+0 (5.12e-3) ≈	1.0826e+0 (9.88e-3) -	1.0669e+0 (9.48e-3)
	10	4.5863e+0 (3.05e-1) -	4.3150e+0 (2.97e-2) ≈	4.4267e+0 (1.13e-1) -	4.5036e+0 (6.60e-2) -	6.6226e+0 (3.57e-1) -	5.7381e+0 (2.82e-1) -	4.3754e+0 (7.00e-2) -	**4.3055e+0 (4.18e-2)**
	15	**8.2927e+0 (5.29e-1) +**	8.3855e+0 (4.38-1) +	8.4757e+0 (5.85e-1) +	8.8589e+0 (1.25e+0) ≈	1.1273e+1 (9.90e-1) -	1.0812e+1 (3.32e-1) -	8.8118e+0 (2.33-1) +	9.2304e+0 (2.26e-1)
	20	1.8290e+1 (1.93e+0) -	1.3351e+1 (4.83e-1) +	1.2971e+1 (1.21e+0) +	1.4130e+1 (1.80e+0) ≈	1.4291e+1 (7.92e-1) ≈	**1.1870e+1 (8.92e-1) +**	1.4335e+1 (2.81e-1) ≈	1.4341e+1 (2.62e-1)
WFG9	5	9.3349e-1 (5.23e-3) ≈	9.2924e-1 (3.55e-3) ≈	9.4331e-1 (3.22e-3) -	9.7647e-1 (1.01e-2) -	1.4060e+0 (1.11e-1) -	1.0344e+0 (4.13e-3) -	**9.2806e-1 (7.16e-3) ≈**	9.3294e-1 (7.87e-3)
	10	4.2950e+0 (5.73e-2) -	4.3041e+0 (3.60e-2) -	4.3115e+0 (6.10e-2) -	4.2827e+0 (4.69e-2) -	5.3797e+0 (1.69e-1) -	5.5712e+0 (3.86e-1) -	3.9773e+0 (2.43e-2) ≈	**3.9651e+0 (2.23e-2)**
	15	7.9971e+0 (1.52e-1) -	7.5770e+0 (1.23e-1) -	7.7724e+0 (2.39e-1) -	7.6602e+0 (1.32e-1) -	9.7553e+0 (2.23e-1) -	8.8861e+0 (1.64e-1) -	7.0467e+0 (1.09e-1) -	**6.9171e+0 (6.36-2)**
	20	1.4364e+1 (7.20e-1) -	1.1896e+1 (1.91e-1) ≈	**1.1754e+1 (3.46e-1) ≈**	1.3285e+1 (7.95e-1) -	1.3362e+1 (2.84e-1) -	1.2939e+1 (2.31e+0) ≈	1.2026e+1 (2.21e-1) ≈	1.1978e+1 (3.38e-1)
+/ − / ≈		10/21/5	15/17/4	14/21/1	5/27/4	0/32/4	6/25/5	11/19/6	

Table 5. IGD results of NSGA-III, θ-DEA, RVEA, NSGAII/SDR, onebyoneEA, MOEA/DD, MultiGPO and NLDEA on MAF1-MAF15 with $G_{\max} = 500$

Problem	M	NSGA-III	θ-DEA	RVEA	NSGAII/SDR	onebyoneEA	MOEADD	MultiGPO	NLDEA
MaF1	5	1.8541e-1 (1.42e-2) -	2.1426e-1 (8.38e-3) -	2.7116e-1 (1.40e-2) -	1.0419e-1 (1.09e-3) +	**1.0178e-1 (1.42e-3) +**	2.0913e-1 (4.33e-3) -	1.0839e-1 (1.30e-3) -	1.0598e-1 (4.91e-4)
	10	2.7865e-1 (4.46e-3) -	3.1756e-1 (1.11e-2) -	5.8061e-1 (8.02e-2) -	**2.1532e-1 (1.29e-3) +**	2.9096e-1 (6. 19e-2) -	4.6726e-1 (1 .93e-2) -	2.3077e-1 (1.53e-3) -	2.2096e-1 (9.73e-4)
	15	3.1799e-1 (9.90e-3) -	3.3730e-1 (9.35e-3) -	6.7168e-1 (5.62e-2) -	2.8222e-1 (4.21e-3) -	4.3531e-1 (3.57e-2) -	5.4334e-1 (3.11e-2) -	3.0793e-1 (1.98e-3) -	**2.7265e-1 (1.09e-3)**
	20	4.4485e-1 (1.24e-2) -	4.5385e-1 (8.55e-3) -	8.3853e-1 (1.0le-1) -	**3.4929e-1 (2.60e-3) +**	5.8593e-1 (2.75e-2) -	6.4501e-1 (3.23e-2) -	4.0204e-1 (2.43e-3) -	3.8939e-1 (1.03e-3)
MaF2	5	1.1192e-1 (2.26e-3) -	1.2418e-1 (3.02e-3) -	1.1581e-1 (8.25e-4) -	9.4872e-2 (1.48e-3) -	**8.2320e-2 (1.84e-3) +**	1.3047e-1 (3.09e-3) -	9.7338e-2 (2.31e-3) -	8.8474e-2 (1 .50e-3)
	10	2.0307e-1 (1.41e-2) -	2.0587e-1 (1.31e-2) -	2.4234e-1 (5.16e-3) -	2.2164e-1 (1.36e-2) -	2.6937e-1 (2.68e-2) -	2.3910e-1 (2.01e-2) -	1.8081e-1 (5.71e-3) -	**1.7153e-1 (2.85e-3)**
	15	2.0924e-1 (6. 76e-3) -	2.7088e-1 (1.86e-2) -	5.4966e-1 (1.51e-1) -	3.5154e-1 (3.63e-2) -	4.7436e-1 (2.59e-2) -	4.0083e-1 (3.86e-2) -	2.0621e-1 (9.45e-3) ≈	**2.0369e-1 (6.06e-3)**
	20	3.2043e-1 (4.24e-2) -	2.8381e-1 (1.54e-2) -	5.7104e-1 (2.10e-1) -	3.6486e-1 (1.72e-2) -	5.6355e-1 (2.13e-2) -	4.6790e-1 (7.15e-2) -	**2.0635e-1 (9.29e-3) +**	2.3283e-1 (1 34e-2)
MaF3	5	**7.7889e-2 (7.04e-3) +**	9.8626e-2 (1.62e-3) -	1.4718e-1 (5.61e-2) -	1.4715e-1 (5.86e-3) -	1.5280e-1 (1.74e-2) -	9.9787e-2 (3.02e-3) -	1.1758e-1 (1.44e-2) -	9.2851e-2 (6.82e-3)
	10	3.0511e+3 (8.85e+3) -	2.6829e-1 (1.41e-1) -	1.2171e-1 (2.11e-2) -	1.5024e-1 (3.30e-3) -	1.2898e-1 (4.90e-2) -	1.8492e-1 (3.85e-1) ≈	2.9993e+1 (8.70e+1) -	**1.0436e-1 (4.66e-3)**
	15	3.3975e+2 (1.06e+3) -	2.4004e-1 (6.54e-2) -	**1.1132e-1 (6.07e-2) +**	1.4097e-1 (1.37e-3) -	6.0169e-1 (1.08e+0) -	1.1862e-1 (4.56e-2) ≈	1.1503e+2 (3.05e+2) -	1.1145e-1 (5.93e-3)
	20	4.1892e+4 (6.06e+4) -	2.0713e+0 (4.11e+0) -	**1.4757e-1 (2.46e-2) +**	1.9857e-1 (9.73e-4) -	2.1171e+0 (4.47e+0) -	2.4697e+1 (7.54e+1) -	1.4795e+2 (2.07e+2) -	1.6614e-1 (8.40e-3)
MaF4	5	2.7846e+0 (1.04e+0) -	3.0606e+0 (4.57e-1) -	3.6608e+0 (1.02e+0) -	2.2393e+0 (9.59e-2) -	5.6549e+0 (8.39e-1) -	5.3102e+0 (5.40e-1) -	1.9111e+0 (5.85e-2) ≈	**1.8872e+0 (6.61e-2)**
	10	9.6209e+1 (6.91e+0) -	1.1068e+2 (9.76e+0) -	1.9935e+2 (3.24e+1) -	1.7767e+2 (2.86e+1) -	2.5169e+2 (3.59e+1) -	3.9796e+2 (1.10e+1) -	6.0081e+1 (5.39e+0) -	**5.1887e+1 (3.10e+0)**
	15	3.8586e+3 (2.94e+2) -	4.4386e+3 (4. 13e+2) -	7.4794e+3 (1.87e+3) -	7.2591e+3 (1.61e+3) -	1.0594e+4 (1.12e+3) -	1.5486e+4 (1.73e+3) -	**1.7472e+3 (1.57e+2) +**	1.9043e+3 (2.18e+2)
	20	1.3542e+5 (1.81e+4) ≈	1.5569e+5 (1.62e+4) ≈	3.1489e+5 (7.00e+4) -	3.1861e+5 (6.27e+4) -	3.8388e+5 (5.12e+3) -	5.5643e+5 (2.48e+4) -	**6.3236e+4 (1.80e+4) +**	1.3927e+5 (4.29e+4)

Table 5. Continued

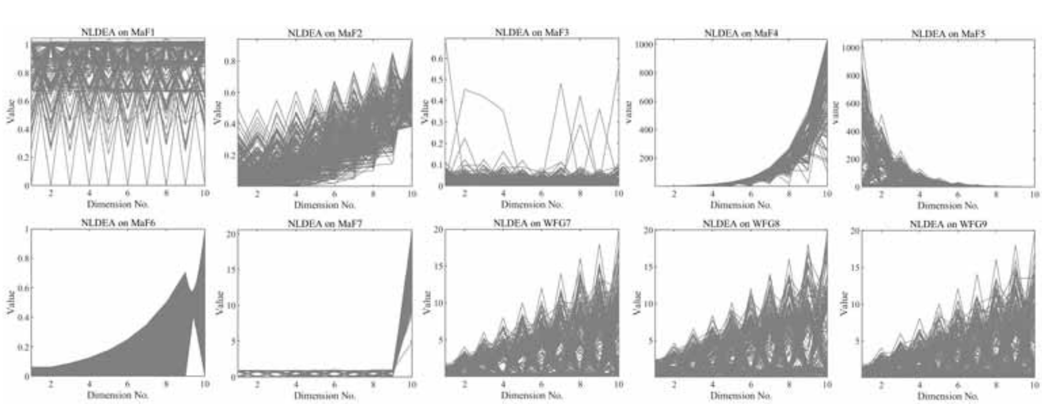
Problem	M	NSGA-III	θ-DEA	RVEA	NSGAII/SDR	onebyoneEA	MOEADD	MultiGPO	NLDEA
MaF5	5	1.9699e+0 (2.68e-3) -	1.9683e+0 (1.86e-3) -	1.9704e+0 (3.47e-3) -	1.2413e+1 (2.71e+0) -	3.7527e+0 (4.96e-1) -	3.9571e+0 (4.88e-1) -	**1.9242e+0 (6.89e-2) +**	1.9452e+0 (8.86e-1)
	10	7.8111e+1 (8.29e-1) -	7.7872e+1 (5.92e-1) -	9.7080e+1 (1.08e+1) -	3.0600e+2 (1.42e+0) -	1.9867e+2 (1.41e+1) -	2.9047e+2 (1.41e+1) -	2.1522e+2 (1.07e+1) -	**5.5659e+1 (2.30e+ 1)**
	15	2.4563e+3 (9.91e+1) -	2.4483e+3 (6.07e+1) -	3.2220e+3 (5.07e+2) -	7.3261e+3 (2.42e-2) -	5. 9906e+3 (9.64e+1) -	7.3002e+3 (4.96e+1) -	5.9955e+3 (1.65e+2) -	**1.8173e+3 (5.56e+2)**
	20	7.4568e+4 (1.71e+4) ≈	7.0018e+4 (6.64e+1) ≈	**6.5929e+4 (2.78e+4) ≈**	1.7095e+5 (1.05e-1) -	1.4156e+5 (2.14e+3) -	1.7090e+5 (8.18e+1) -	1.3532e+5 (1 .59e+4) -	7.4732e+4 (2.13e+4)
MaF6	5	1.6847e-2 (5.17e-3) -	8.5216e-2 (1.41e-2) -	7.2969-2 (7.80e-3) -	1.5610e-2 (1.33e-2) -	2.1096e-3 (3.88e-5) -	6.5701e-2 (4.80e-3) -	3.4378e-3 (2.37e-4) -	**2.0592e-3 (1.90e-5)**
	10	7.1234e-1 (3.02e-1) -	9.0433e-2 (7.79e-2) -	1.2193e-1 (1.74e-2) -	1.1906e-2 (9.31e-3) -	1.6031e-3 (2.25e-5) -	9.0452e-2 (1.55e-2) -	1.0498e-1 (1.61e-1) -	**1.4829e-3 (5.39e-6)**
	15	7.8246e-1 (2.80e-1) -	3.8988e-1 (1.26e-1) -	5.1757e-1 (2.44e-1) -	1 9506e-2 (6.67e-2) -	1.8420e-3 (2.71e-5) -	1.2225e-1 (8.75e-3) -	3.9379e-1 (7.73e-2) -	**1.7346e-3 (1.97e-5)**
	20	6.8526e+0 (8.59e+0) -	7.4932e-1 (4.43e-1) -	1.9669e-1 (2.00e-2) -	9.8692e-2 (1.34e-1) -	2.0993e-3 (3.09e-5) -	1.7016e-1 (1.92e-2) -	4.4019e-1 (1.30e-1) -	**2.061le-3 (1.56e-5)**
MaF7	5	2.8099e-1 (7.29e-3) -	3.0065e-1 (2.73e-2) -	5.0195e-1 (8.92e-3) -	3.2404e-1 (2.36e-2) -	3.2340e-1 (3.61e-2) -	2.4246e+0 (1.03e+0) -	2.8322e-1 (1.16e-1) -	**2.5488e-1 (5.33e-3)**
	10	1.0769e+0 (7.21e-2) -	9.6354e-1 (1.02e-1) -	2.1802e+0 (4.93e-1) -	1.5573e+0 (2.50e-1) -	2.1958e+0 (4.89e-1) -	2.5715e+0 (4.46e-1) -	8.8740e-1 (1.55e-2) -	**8.7623e-1 (1.02e-2)**
	15	4.1741e+0 (4.94e-1) -	3. 7658e+0 (5.26e-1) -	3.1406e+0 (5.96e-1) -	4.1605e+0 (4.00e-1) -	3.1379e+0 (4.32e-1) -	3.4541e+0 (5.12e-2) -	2.0542e+0 (1.27e-1) -	**1.9080e+0 (9.48e-2)**
	20	8.3123e+0 (8.72e-1) -	9.6452e+0 (9.34e-1) -	7.0275e+0 (1.68e+0) -	7.3432e+0 (5.59e-1) -	3.6696e+0 (2.22e-1) -	1.5633e+1 (7.48e-1) -	4.1594e+0 (8.69e-1) -	**2.7233e+0 (2.75e-1)**
MaF8	5	1.5849e-1 (9.67e-3) -	3.0751e-1 (4. 42e-2) -	3.1415e-1 (3.20e-2) -	9.7832e-2 (4.56e-3) -	3.6022e-1 (6.96e-2) -	2.7976e-1 (1.83e-2) -	**7.7308e-2 (9.02e-4) +**	8.5008e-2 (4.98e-3)
	10	3.1987e-1 (6.91e-2) -	7.1820e-1 (1.04e-1) -	9.7509e-1 (1.44e-1) -	1.4337e-1 (8.80e-3) -	3.4119e-1 (7.07e-2) -	9.0351e-1 (3.77e-2) -	**1.0295e-1 (1.08e-3) +**	1.1759e-1 (5.23e-3)
	15	4.0866e-1 (1.04e-1) -	9.4423e-1 (1.79e-1) -	1.2713e+0 (1.89e-1) -	2.0245e-1 (1.77e-2) -	4.0401e-1 (6.70e-2) -	1.3194e+0 (4.36e-2) -	**1.3637e-1 (1.49e-3) +**	1.6088e-1 (1.12e-2)
	20	4.5365e-1 (5.44e-2) -	9.8686e-1 (2.67e-1) -	1.4410e+0 (2.04e-1) -	2.6280e-1 (3.28e-2) -	4.6715e-1 (6.90e-2) -	1.8494e+0 (8.18e-2) -	**1.7471e-1 (2.49e-3) +**	2.0487e-1 (9.76-3)
MaF9	5	4.1393e-1 (1.75e-1) -	6.1091e-1 (1.91e-1) -	2.8636e-1 (4.51e-2) -	1.3419e-1 (5.63e-3) -	1.4163e-1 (3.40e-2) -	2.2603e-1 (1.64e-3) -	**7. 1694e-2 (6.05e-4) +**	8.3369e-2 (2.42e-2)
	10	5.3683e-1 (1.01e-1) -	8.1366e-1 (1.46e-1) -	8.4440e-1 (2.0le-1) -	1.6966e-1 (6.51e-3) -	1.1317e-1 (6.74e-3) -	5.9367e-1 (4.65e-3) -	**9.7474e-2 (6.23e-4) +**	1.0650e-1 (5.69e-3)
	15	3.7716e-1 (6.54e-2) -	9.1240e-1 (2.98e-1) -	1.3655e+0 (3.00e-1) -	1.9102e-1 (4.84e-3) -	2.5257e-1 (2.48e-1) -	9.5699e-1 (1.53e-2) -	**1.2965e-1 (6.86e-4) +**	1.9050e-1 (1.90e-1)
	20	1.3550e+1 (8.76e+0) -	5.0657e+0 (6.53e+0) -	1.4994e+0 (2.16e-1) -	2.3142e-1 (6.03e-3) +	2.9612e-1 (1.22e-1) ≈	2.3867e+0 (3.13e+0) -	**1.6235e-1 (2.12e-3) +**	3.4527e-1 (6.48e-1)
MaF10	5	3.7112e-1 (5.84e-3) +	**3.6235e-1 (5.24e-3) +**	3.7893e-1 (9.11e-3) +	6.8762e-1 (1.13e-1) -	7.0516e-1 (3.00e-2) -	4.6000e-1 (2.56e-2) +	3.7744e-1 (1.03e-2) +	5.0099e-1 (1.56e-2)
	10	1.0268e+0 (5.95e-2) +	**9.8369e-1 (1.58e-2) +**	1.0877e+0 (3.95e-2) +	1.7852e+0 (1.20e-1) -	1.7947e+0 (3.97e-2) -	1.3634e+0 (7.63e-2) -	1.0380e+0 (1.13e-1) +	1.1504e+0 (4.46e-2)
	15	1.5572e+0 (1.02e-1) +	**1.5070e+0 (2.90e-2) +**	1.6264e+0 (6.31-2) +	2.4500e+0 (3.92e-2) -	2.4286e+0 (2.33e-2) -	1.9891e+0 (6.56e-2) -	1.6923e+0 (1.53e-1) +	1.8279e+0 (7.91e-2)
	20	4.3742e+0 (4.69e-1) ≈	4. 4567e+0 (4.35e-1) -	4.0512e+0 (1.08e-1) +	5.1637e+0 (3.42e-2) -	5.1763e+0 (2.47e-2) -	5.2536e+0 (2.24e-2) -	**3.8159e+0 (3.54e-1) +**	4.2422e+0 (2. 42e-1)
MaF11	5	3.8900e-1 (1.37e-3) +	3.8890e-1 (3.03e-3) +	**3.8520e-1 (9.37e-3) +**	5.1186e-1 (4.52e-2) ≈	6.6517e-1 (5.36e-2) -	5.1174e-1 (1.25e-2) ≈	4.7013e-1 (9.11e-2) +	5.3708e-1 (6.19e-2)
	10	1.2755e+0 (8.19e-2) +	1.1312e+0 (5.51e-2) +	**1.0969e+0 (2.70e-2) +**	1.5711e+0 (1.01e-1) +	1.8395e+0 (5.57e-2) -	1.4733e+0 (1.59e-2) +	1.4438e+0 (7.14e-2) +	1.6438e+0 (7.62e-2)
	15	**1.5354e+0 (5.63e-2) +**	4.6742e+0 (1.35e+0) -	1.7029e+0 (7.29e-2) +	2.3591e+0 (6.86e-2) -	2.3651e+0 (7.16e-2) -	1.9186e+0 (4.99e-2) +	2.2176e+0 (9.43e-2) ≈	2.2415e+0 (1.26e-1)
	20	4.0010e+0 (1.43e-1) +	4.5103e+0 (3.84e-1) +	**3.4249e+0 (2.69e-1) +**	5.1872e+0 (6.81e-2) -	4.9955e+0 (1.62e-1) -	5.5366e+0 (1.35e-2) -	4.5824e+0 (2.15e-1) +	4.8120e+0 (2. 19e-1)

continued on following page

Table 5. Continued

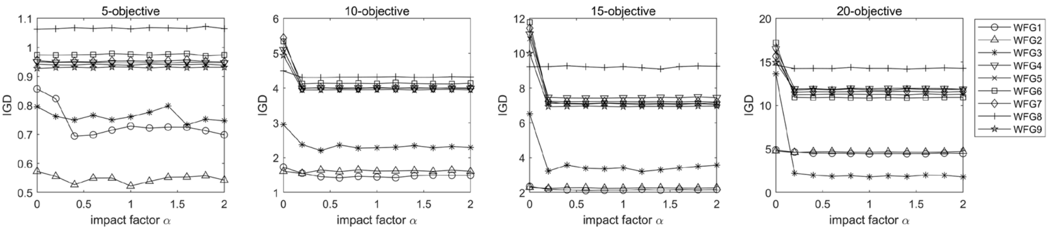
Problem	M	NSGA-III	θ-DEA	RVEA	NSGAII/SDR	onebyoneEA	MOEADD	MultiGPO	NLDEA
MaF12	5	9.3559e-1 (2.08e-3) -	9.3206e-1 (3.13e-3) -	9.4387e-1 (1.45e-3) -	9.7513e-1 (8. 10e-3) -	1.4219e+0 (9.39e-2) -	1.0319e+0 (5.70e-3) -	**9.2455e-1 (7.12e-3) ≈**	9.2676e-1 (6.16e-3)
	10	4.4051e+0 (2.24e-2) -	4.3874e+0 (2.96e-2) -	4.2374e+0 (4.74e-2) -	4.3188e+0 (2.95e-2) -	5.4350e+0 (1.73e-1) -	6.2793e+0 (2.46e-1) -	3.9881e+0 (2.82e-2) -	**3.9418e+0 (2.48e-2)**
	15	7.9275e+0 (1.39e-1) -	7.5877e+0 (1.49e-1) -	7.4459e+0 (3.50e-1) -	7.6645e+0 (1.50e-1) -	9.8567e+0 (3.08e-1) -	8.6103e+0 (1.13e-1) -	7.0029e+0 (8.85e-2) -	**6.8823e+0 (6.79e-2)**
	20	1.3804e+1 (8.48e-1) -	**1. 1362e+1 (1.46e-1) ≈**	1.2223e+1 (4.27e-1) -	1.2822e+1 (1.52e+0) -	1.3614e+1 (3.72e-1) -	1.5836e+1 (3.20e+0) -	1.1637e+1 (1.84e-1) -	1.1434e+1 (2.04e-1)
MaF13	5	2.0715e-1 (2.31e-2) -	3.0154e-1 (4.98e-2) -	3.6382e-1 (4.52e-2) -	1.3911e-1 (1.05e-2) -	**8.1727e-2 (3.93e-3) +**	2.0086e-1 (1.54e-2) -	9.3789e-2 (2.01e-2) ≈	1.0168e-1 (9.54e-3)
	10	2.4097e-1 (2.08e-2) -	5.9662e-1 (6.65e-2) -	8.0155e-1 (1.84e-1) -	1.6769e-1 (1.21e-2) -	1.2714e-1 (1.66e-2) ≈	3.5365e-1 (3.69e-2) -	**1.1898e-1 (3.40e-2) ≈**	1.2023e-1 (4.87e-3)
	15	2.9155e-1 (3.70e-2) -	6.9143e-1 (9.77e-2) -	1 2066e+0 (4.86e-1) -	1.7187e-1 (1.14e-2) -	2.3711e-1 (5.49e-2) -	3.7704e-1 (3.25e-2) -	2.9059e-1 (3.93e-2) -	**1.6233e-1 (9.96e-3)**
	20	3.0198e-1 (2.46e-2) -	1.2010e+0 (3.00e-1) -	1.0195e+0 (2.34e-1) -	**1.7820e-1 (1.16e-2) +**	3.2228e-1 (9.54e-2) -	3.9952e-1 (3.80e-2) -	4.0046e-1 (4.50e-2) -	1.9464e-1 (1 52e-2)
MaF14	5	1.7678e+0 (1.19e+0) -	1.5900e+0 (1.16e+0) -	9.2527e-1 (1.62e-1) -	5.4829e-1 (1.21e-1) -	4.7642e-1 (9.61e-2) ≈	7.1373e-1 (1.22e-1) -	4.9798e-1 (4.84e-2) -	**4.6583e-1 (4.47e-2)**
	10	1.0779e+1 (6.47e+0) -	7.4966e+0 (3.94e+0) -	1.0572e+0 (5.71e-2) -	1.1130e+0 (1.31e-1) -	1.1649e+0 (1.69e-1) -	1.1935e+0 (2.00e-1) -	**7.8505e-1 (1.06e-1) ≈**	8.3407e-1 (1.20e-1)
	15	3.6909e+0 (4.28e+0) -	3.1404e+0 (4.00e+0) -	2.2374e+0 (9.95e-1) -	1.2677e+0 (2.28e-1) ≈	2.9733e+0 (1.39e+0) -	1.3196e+0 (2.44e-1) -	1.4955e+0 (5.97e-1) ≈	**1.1709e+0 (2.73e-1)**
	20	4.1403e+0 (4.89e+0) -	4.6249e+0 (2.98e+0) -	1.0888e+0 (5.80e-2) ≈	1.0682e+0 (1.72e-2) ≈	1.1357e+0 (1.45e-1) ≈	1.2013e+0 (1.91e-1) -	**9.6523e-1 (1.63e-1) +**	1.0771e+0 (9.54e-2)
MaF15	5	1.1652e+0 (134e-1) -	1.1732e+0 (1.03e-1) -	5.2957e-1 (4.12e-2) +	7.9559e-1 (2.66e-2) -	5.9090e-1 (7.33e-2) +	**4.8802e-1 (2.79e-2) +**	8.5646e-1 (3.65e-2) -	6.9238e-1 (6.55e-2)
	10	2.1917e+0 (7.87e-1) -	1.6582e+0 (4.79e-1) -	**1.0550e+0 (6.00e-2) +**	1.0812e+0 (2.39e-2) +	1.1120e+0 (4.50e-2) ≈	1.0640e+0 (4.84e-2) +	1.1390e+0 (6.66e-2) -	1.1117e+0 (3.49e-2)
	15	1.0687e+1 (4.30e+0) -	4.2861e+0 (1.68e+0) -	1.2671e+0 (5.66e-2) +	**1.2629e+0 (3.69e-2) +**	1.4878e+0 (8.72e-2) -	1.4926e+0 (1.70e-1) -	1.3972e+0 (3.70e-2) -	1.3145e+0 (4.12e-2)
	20	1.8457e+1 (6.80e+0) -	8.7118e+0 (3.30e+0) -	1.4634e+0 (5.59e-2) +	**1.4278e+0 (4.79e-2) +**	2.0833e+0 (2.14e-1) -	3.0145e+0 (1.10e+0) -	1.6114e+0 (1.05e-1) ≈	1.5583e+0 (1.12e-1)
+/ − / ≈		8/49/3	6/51/3	14/44/2	9/48/3	4/51/5	5/52/3	20/31/9	

Figure 3. Parallel coordinates of the objective values for each algorithm on ten-objective maf problems and WFG problems



From Figure 5, we can see that on the WFG test suite with 5-objective, the value of PD increases as the value of α increases for NLDEA, but at the same time, the value of GD increases. The value of GD reflects its convergence; the lower the value of GD, the better the convergence performance,

Figure 4. IGD results of NLDEA on WFG problems with different settings of α for different numbers of objectives



while the value of PD reflects the ability of diversity, the higher the value of PD, the better the diversity performance.

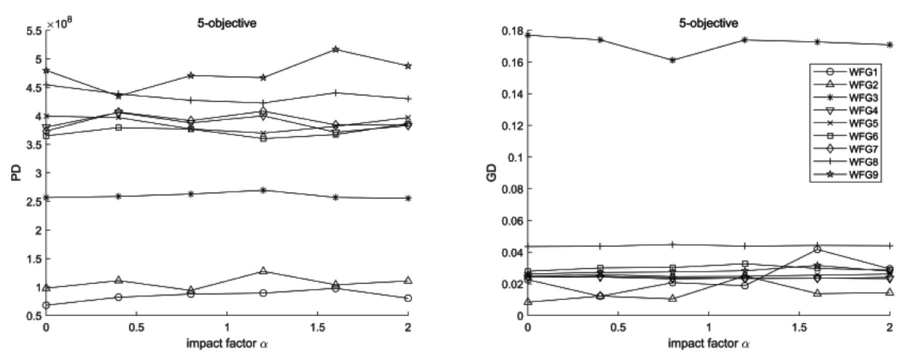
Application to Real-World Problems

In this part, the effectiveness of NLDEA was further verified in the practical problem of unconstrained and irregular PF, that is, water and oil repellent fabric development (WORFD) (Ahmad et al., 2017; Zapotecas-Martínez et al., 2023). The textile industry aims to create high-value fabrics, often achieving hydrophobicity, or water and oil repellency, through various process parameters (Sun et al., 2005). To assess hydrophobicity, seven criteria can be used (Ahmad et al., 2017): water and oil droplet contact angle (WCA and OCA), air permeability (AP), crease recovery angle (CRA), stiffness (Stiff), tear strength (Tear), and tensile strength (Tensile). These criteria serve as objective functions. WCA and OCA measure droplet contact angle, AP assesses airflow through the fabric, CRA measures the ability to recover from creases, Stiff describes cotton fabric comfort, Tear depends on the finishing treatment, and Tensile evaluates behavior under axial stretching.

NLDEA is compared with MultiGPO in terms of performance metrics HV. The parameters α and φ for NLDEA and MultiGPO are set to 1.1 and 21, respectively. For all methods, the population size was set to 210 and the maximum number of generations was 200. Afterwards, the HV values were computed with reference point $\left(1.1,\ldots,1.1\right)^T$.

As can be seen from Figure 6, the performance of NLDEA and MultiGPO is comparable or even better. The larger the HV value, the better the convergence and diversity of the algorithm.

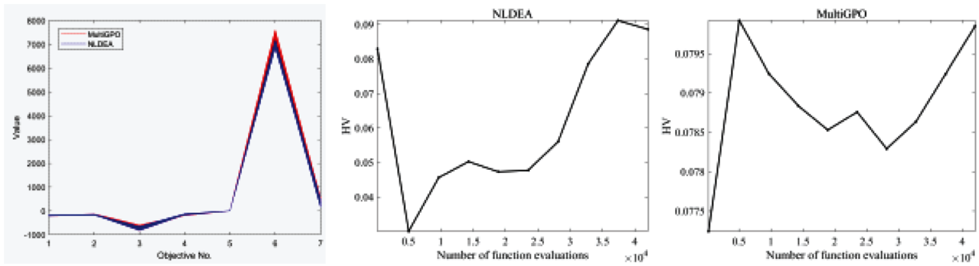
Figure 5. Effect of setting different α Values on PD and GD in NLDEA for WFG problems with 5-Objective



(a) PD in NLDEA for WFG problems with 5-objective

(b) GD in NLDEA for WFG problems with 5-objective

Figure 6. The results of NLDEA and MultiGPO on WORFD



(a) WORFD values of objectives (b) HV of NLDEA on WORFD (c) HV of MultiGPO on WORFD

CONCLUSION

In this paper, a new nonlinearly dominated multi-objective optimization evolutionary algorithm is proposed. The algorithm uses the NLD framework for solution ranking, which is simple to implement and maintains a good balance of convergence and diversity. The EMO framework based on NLD environment selection, i.e., NLDEA, has shown superior performance when compared to existing techniques on various scalable benchmark problems and real-world problems. Since no reference vector is used, the performance of NLDEA is less dependent on the PF shape and is robust compared to methods using reference vectors, especially when solving problems with irregular PFs. However, the proposed NLD scheme can still be combined with reference vector-based techniques, especially those that use reference vector adaptation, to achieve further performance improvements on certain problems. These findings suggest that there is still untapped potential in the NLD framework, and further research is needed to fully explore its capabilities.

CONFLICT OF INTEREST

All authors of this article declare there are no competing interests.

FUNDING STATEMENT

This work was supported by National Natural Science Foundations of China [no. 12271326, no. 61806120, no. 61502290, no. 61401263, no. 61672334, no. 61673251]; China Postdoctoral Science Foundation [no. 2015M582606]; Industrial Research Project of Science and Technology in Shaanxi Province [no. 2015GY016, no. 2017JQ6063]; Fundamental Research Fund for the Central Universities [no. GK202003071]; and Natural Science Basic Research Plan in Shaanxi Province of China [no. 2022JM-354].

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