

Multiple Criteria DEA-Based Ranking Approach With the Transformation of Decision-Making Units

Jae-Dong Hong, South Carolina State University, USA

ABSTRACT

Though various ranking methods in the data envelopment analysis (DEA) context have emerged since the conventional DEA was introduced, none of them has not been accepted as a universal or a superior method for ranking decision-making units (DMUs). The DEA-based ranking methods show some shortcomings as the numbers of inputs and outputs for DMUs increase. To overcome such shortcomings, this paper proposes a two-step procedure of ranking DMUs more effectively and consistently. In the first step, the multi-objective programming (MOP) is applied for the multiple criteria DEA to transform the original DMUs into the new simpler DMUs with two inputs and a single output, regardless of the numbers of inputs and outputs that the original DMUs use and produce. With the transformed DMUs, some conventional DEA-based methods for ranking DMUs are applied in the second step. A numerical example demonstrates the efficient performance of the proposed method.

KEYWORDS

Conventional DEA, Data Envelopment Analysis (DEA), Decision-Making Unit, Multiple-Criteria DEA, Multi-Objective Programming Model, Ranking Method

INTRODUCTION

The conventional data envelopment analysis was first introduced in 1978 by Charnes, Cooper, and Rhodes (1978), in evaluating the efficiency of a set of peer organizations called decision-making units (DMUs) that consume multiple inputs to generate various outputs. DEA methods have been widely accepted as an effective technique in identifying and separating efficient DMUs from inefficient ones. But the conventional DEA intrinsically aims to identify efficient DMUs and the efficient frontier, so the use of DEA is not enough for discriminating between efficient DMUs. In terms of ranking DMUs, many authors show that conventional DEA is not an appropriate method in many situations. Consequently, the researchers and practitioners have faced a question, “Which DEA method should we use for ranking DMUs effectively and consistently?” Publication and research work have grown substantially, resulting in significant advancements in its methodologies, models, and real-world applications (see Cook and Seiford, 2009; Chen et al., 2019).

DOI: 10.4018/IJAIE.20210101.0a2

This article published as an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0/>) which permits unrestricted use, distribution, and production in any medium, provided the author of the original work and original publication source are properly credited.

Charnes et al. (1978) demonstrate how to change a fractional linear measure of efficiency into a linear programming (LP) format to measure efficiency scores (ESs) of DMUs. In the conventional DEA (C-DEA), a relative efficiency is defined as the ratio of the sum of weighted outputs to the sum of weighted inputs. The C-DEA solves an LP formulation for each DMU to be rated, and the weights assigned to each linear aggregation are obtained by solving the corresponding LP. The DMUs in the C-DEA to be assessed should be relatively homogeneous. As the whole technique is based on a comparison of each DMU with all the remaining ones, a considerable large set of DMUs is necessary for the assessment to be meaningful (Ramanathan, 2006). The C-DEA eventually determines which of the DMUs make efficient use of their inputs and produce most outputs and which DMUs do not. The significant function that the conventional DEA model can do is to separate efficient DMUs from inefficient DMUs. For the inefficient DMUs, the analysis can quantify what levels of improved performance should be attainable. Also, the study indicates where an inefficient DMU might look for benchmarking help as it searches for ways to improve. Recently, Cao et al. (2020) introduce the concept of the anti-strike ability of a single DMU and provide a new ranking method of DMUs. Shahghobadi (2020) presents a method for performance assessment of units so that a large number of units are not evaluated as efficient, but there is at least one efficient unit. Toloo et al. (2020) contend that the number of performance factors (inputs and outputs) plays a decisive role when applying DEA to real-world applications.

The C-DEA produces a single, complete measure of performance for each DMU. The highest efficiency score among all the DMUs would identify the most efficient DMU(s), and every other DMU would be evaluated by comparing its ratio to the DMU with the highest one. A significant weakness of the C-DEA-based assessment comes out because a considerable number of DMUs out of the set of DMUs to be rated can be classified as efficient, and all efficient DMUs are considered to be equal. The nature of the self-evaluation of C-DEA allows each DMU to be evaluated with its most favorable weights. Thus, to maximize the self-efficiency, the conventional DEA model intentionally ignores unfavorable inputs/outputs. The DEA-based methods are developed to measure the relative efficiency of DMUs with multiple inputs and outputs. If DMUs have a single output and a single input, any DEA ranking method is necessary. As the number of inputs and/or outputs of DMUs to be rated increases, the weaknesses of DEA-related methods become more apparent since the DEA method can overlook the weights assigned to unfavorable inputs or outputs

Since the weakness of C-DEA results from its pure self-evaluation, a DEA extension is suggested by Sexton et al. (1986), which is called the cross-efficiency (CE) DEA method. The CE-DEA with the main idea of using the conventional DEA to add the peer evaluation to the pure self-evaluation enhances the discrimination power, and the efficient DMUs treated equally by the conventional DEA can be ranked by their cross-efficiency scores (CESSs). Sexton et al. (1986) construct a CE matrix consisting of two rating results, the self-evaluation and the peer-evaluation. The CE-DEA can provide a full ranking for the DMUs to be evaluated and eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application area experts (see Anderson et al., 2002). Due to its enhanced discriminating power, especially for the simple DMUs with few inputs and outputs, There are a significant number of applications using the CE evaluation in the DEA literature (see Gavgani and Zohrehbandian, 2014; Hou et al., 2018; Lee, 2019; Liang et al., 2009; Liu et al., 2019; Wang and Chin, 2010).

There have been some crucial issues facing the CE method application. The first issue is the ratio of self-evaluation to peer-evaluation in computing the CES. Doyle and Green (1994) exclude the proportion of self-evaluation by eliminating the diagonal elements in the CE matrix to compute CESSs. Some researchers suggest that the percentage of self-evaluation be $1/N$, where N is the total number of DMUs to be evaluated. The second issue is that the non-uniqueness of CESSs due to the often-present multiple optimal DEA weights. It implies that the CES produced by the CE method is not unique, but flexible, depending on the optimization software used. They (1994) suggest that secondary goals such as aggressive and benevolent models for the CE evaluation. Later, Wang and

Chin (2010) propose a neutral CE model that determines one set of input and output weights for each DMU without being aggressive or benevolent to the others. Thus, the resulting CESs would be more neutral.

Li and Reeves (1999) propose multiple criteria (MC) model under the framework of multiple objective linear programming (MOLP). The DEA model involves a broader definition of relative efficiency than conventional DEA. In other words, three different efficiency measures, which is to be maximized or minimized, are defined under the same constraints. They (1999) claim that efficiency criteria that are more restrictive than the DEA one will yield fewer efficient DMUs and will allow less flexibility for input/output weight distribution. However, there is no way to rank efficient DMUs if multiple DMUs turn out to be efficient. They (1999) merely show which DMUs are more consistently efficient than other DMUs by solving the DEA sequentially with one objective out of three objective functions. The question of how to rank efficient DMUs in terms of efficiency remains to be answered.

Anderson and Peterson (1993) mainly propose the super-efficiency (SE) method, whose central idea is that a DMU under evaluation is not included in the reference set of the conventional DEA models, and then with its inclusion. The resulting super-efficiency scores (SEs) are used to rank the efficient DMUs with ES of 1 generated by the conventional DEA. Notably, the SE method has significance for discriminating among efficient DMUs. See also Nayeibi and Lotfi (2016) and Deng et al. (2018) for further applications of the SE method. Guo et al. (2017) and Tran et al. (2019) develop an integrated model for slacks-based measure simultaneously of both the efficiency and the SE for DMUs. But the critical issue of using the model is that the SE score (SES) of an efficient DMU is decided by the adjacent DMUs, so it would be unreasonable for DMUs to be ranked by the SESs.

As mentioned before, these DEA methods generally tend to show their weaknesses. This paper proposes a new procedure of transforming DMUs by multi-objective programming (MOP) approach so that DMUs with multiple inputs and/or outputs are transformed into the simpler DMUs with two inputs and one output. Then, those DEA methods are integrated for evaluating DMUs to generate more robust and consistent rankings.

LITERATURE REVIEW

There have been several review papers on ranking methods in the DEA context as an attempt to fill the gap in the literature of DEA ranking methods, such as Adler et al. (2002), Jahanshahloo et al. (2008), Markovits-Somogyi (2011), Lotfi et al. (2013), and Aldamak and Zolfaghari (2017). These review papers with titles and the number of references reviewed by each paper are listed chronologically in Table 1. The methods or models that each paper reviews and the major findings of each paper are also briefly described. These papers reveal that there have existed a variety of papers that apply different ranking methods, seeking to improve the discriminating power of DEA methods and to fully rank all DMUs whether they turn out to be efficient or not.

The common ranking methods/models reviewed by these review papers are cross efficiency, super efficiency, benchmarking, multi-criteria decision making, and statistical analysis. Adler et al. (2002) conclude that many mathematical and statistical techniques have been presented, all with the objective of increasing the discriminating power for the DEA-based methods and ranking the DMUs. But, while each method may be useful in a specific area, no one methodology can be prescribed as the panacea of all ills. They (2002) expect the ultimate DEA model to be developed to solve all weaknesses or problems and to be easy to solve by practitioners in the field and academics alike. After 15 years since Adler et al. (2002) expect the ultimate DEA model to be developed as a conclusion, Aldamak and Zolfaghari (2017) find that none of the proposed DEA ranking categories is optimum for every evaluation assessment. They review one hundred and twelve (112) articles published in various scholarly journals with the subject of DEA-based ranking methods and conclude that no ranking method has been found to be either a universal or a superior method for ranking the efficiency of DEA models. They also say that the absence of global assessment criteria makes it impossible to

evaluate all the presented methods reviewed by their paper. They insist that each method could be better than others according to the decision-makers' preferences and evaluation objectives, which depends on the nature of the evaluation.

As mentioned before, each method mentioned in the previous section has shown its own weaknesses. In fact, conventional DEA-based methods sometimes rank efficient DMUs very differently (see Hong and Jeong, 2017). Decision-makers usually are interested in selecting the top-ranked DMU(s) before they make final decisions of selecting the most efficient DMUs among efficient ones. If the #1 ranked DMU by the CE method is ranked very low by other methods, it would confuse the decision-makers or practitioners. The proposed method might not be a universal or a superior method, which can answer the question, 'which DEA ranking method one should use?'. However, it attempts to eliminate or at least weaken each method's weakness by transforming the original DMUs into new DMUs with a single output and two inputs, regardless of the numbers of inputs and outputs that the original DMUs have. If the proposed method as well as the other DEA methods can rank the DMU as #1 consistently after the inefficient DMUs from the whole group of DMUs to be rated are removed, it would be an ideal case for the proposed method to be considered a robust ranking method.

Table 1. Summary of the review papers related to the ranking methods in DEA

Authors	Title	# of Reviewed References	Reviewed Ranking Methods/Models	Major Findings
Adler et al. (2002)	"Review of ranking methods in the data envelopment analysis context"	59	Cross efficiency, Super efficiency, Benchmarking, Multivariate statistics, Proportional measures, Multi-criteria decision-making method	Many mathematical and statistical techniques have been presented to increasing the discriminating power of DEA and fully rank the DMUs.
Jahanshahloo et al. (2008)	"Review of ranking models in the data envelopment analysis"	22	Super efficiency-based AP, MAJ, Revised MAJ, Slack adjusted, Gradient line models, Using Common set of weights, L1 Norm, Concept of advantage model, Tchebycheff Norm. Monte Carlo method and Slack based model (SBM).	Some models are infeasible for special data. Monte Carlo method is suggested to rank all efficient DMUs because it ranks extreme and non-extreme DMUs.
Markovits-Somogyi (2011)	"Ranking efficient and inefficient decision-making units in data envelopment analysis"	43	Cross efficiency, Super efficiency, Benchmarking, Minimum weight restriction, Statistical analysis, Slack based DEA, Multi-criteria decision-making methods, Application of fuzzy logic, Shadow prices.	The full ranking is achievable through several ways in DEA, and the choice of a specific method would depend on the particular needs of the study in question.
Lotfi et al. (2013)	"A review of ranking models in data envelopment analysis"	104	Cross efficiency, Super efficiency, Optimal weights in DEA, Benchmarking, Multivariate Statistics, Multi-criteria decision-making method, Stratification, Gradient line.	The DEA ranking is reviewed and classified into seven general groups. Some of the reviewed models are applied to the example, and none of the DMUs is ranked consistently.
Aldamak and Zolfaghari (2017)	"Review of efficiency ranking methods in data envelopment analysis"	112	Cross efficiency, Super efficiency, Benchmarking, Statistics, Common weights, Multi-criteria decision-making method, Inefficient frontier, Virtual DMU, DM interference, Fuzzy concept.	The absence of universal assessment criteria makes it impossible to evaluate the presented methods. Each method could be better than others, according to decision-makers' preferences and evaluation objectives.

TRANSFORMATION OF DMUS

Charnes et al. (1978) establish a CRS (Constant Returns to Scale) m-DEA model to find an efficiency score (ES) for DMU_k , which is formulated as the following LP problem:

$$\max \theta_k = \sum_{r=1}^s u_r y_{rk} \quad (1)$$

subject to:

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (2)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \forall j = 1, 2, \dots, N \quad (3)$$

$$u_r, v_i \geq 0, r = 1 \dots, s; i = 1 \dots, m$$

where:

N = number of DMUs to be rated in the DEA analysis

θ_k = efficiency rating of the DMU_k being evaluated by DEA

y_{rj} = amount of output r produced by DMU_j , $j = 1, 2, \dots, k, \dots, N$

x_{ij} = amount of input i used by DMU_j

i = number of inputs utilized by the DMUs

r = number of outputs produced by the DMUs

u_r = coefficient or weight assigned by DEA to output r

v_i = coefficient or weight assigned by DEA to input i .

The MC DEA model is proposed by Li and Reeves (1999), using the LP model given in (1)-(3):

$$\max \theta_k = \sum_{r=1}^s u_r y_{rk} \quad (4)$$

$$\min D_k = \sum_j d_j \quad (5)$$

$$\min M_k (= \max d_j) \quad (6)$$

subject to:

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (7)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + d_j = 0, \forall j \quad (8)$$

$$M_k - d_j \geq 0, \forall j \quad (9)$$

$$u_r, v_i \geq 0, r = 1, \dots, s; i = 1, \dots, m; \text{ and } d_j \geq 0, j = 1, 2, \dots, k, \dots, N$$

There are three performance criteria in the MC DEA model, θ_k , ES in (4), M_k given in (6) represents the maximum quantity among all deviation variables, d_j , and D_k given in (5) is equivalent to the sum of deviation variables. Solving the MC model sequentially with one objective out of three objective functions. The DMUs, whose efficiency score, θ_k , remains the maximum value of 1 regardless of the objective function used, are considered more efficient than other DMUs. Similar to conventional DEA, the MC method treats all efficient DMUs equally. The advantage of the MC method over the conventional DEA is MC method would select less efficient DMUs than the conventional DEA.

To rank the evaluated DMUs effectively, this paper proposes the following procedure (see Ragsdale, 2017). Let the nonnegative deviation variables, $\delta_{\theta(k)}^-$, $\delta_{M(k)}^+$ and $\delta_{D(k)}^+$, denote the amounts by which each value of θ_k , D_k , and M_k deviates from the maximum value of θ_k , which is equal to 1, and minimum values of D_k and M_k , respectively.

$$\delta_{\theta(k)}^- = 1 - \theta_k \quad (10)$$

$$\delta_{D(k)}^+ = D_k - \min \{D_j, \forall j\} \quad (11)$$

$$\delta_{M(k)}^+ = M_k - \min \{M_j, \forall j\} \quad (12)$$

Now, the following MOP for the MC DEA model is formulated:

$$\text{Minimize } W \quad (13)$$

subject to Constraints (7)-(12):

$$\alpha_1^- \delta_{h(k)}^- \leq W \quad (14)$$

$$\alpha_2^+ \frac{\delta_{M(k)}^+}{\min \{M_j, \forall j\}} \leq W \quad (15)$$

$$\alpha_3^+ \frac{\delta_{D(k)}^+}{\min \{D_j, \forall j\}} \leq W \quad (16)$$

$$\alpha_1^- + \alpha_2^+ + \alpha_3^+ = 1 \quad (17)$$

Solving the above MOP for a given set of $\alpha = \{\alpha_1^-, \alpha_2^+, \alpha_3^+\}$ yields one alternative with the optimal values of three performance measures. Thus, there will be many alternatives generated by solving the MOP model for various sets of α . But the question remains how the DMUs are ranked after various alternatives are generated by solving the above MOP model with different set values of α .

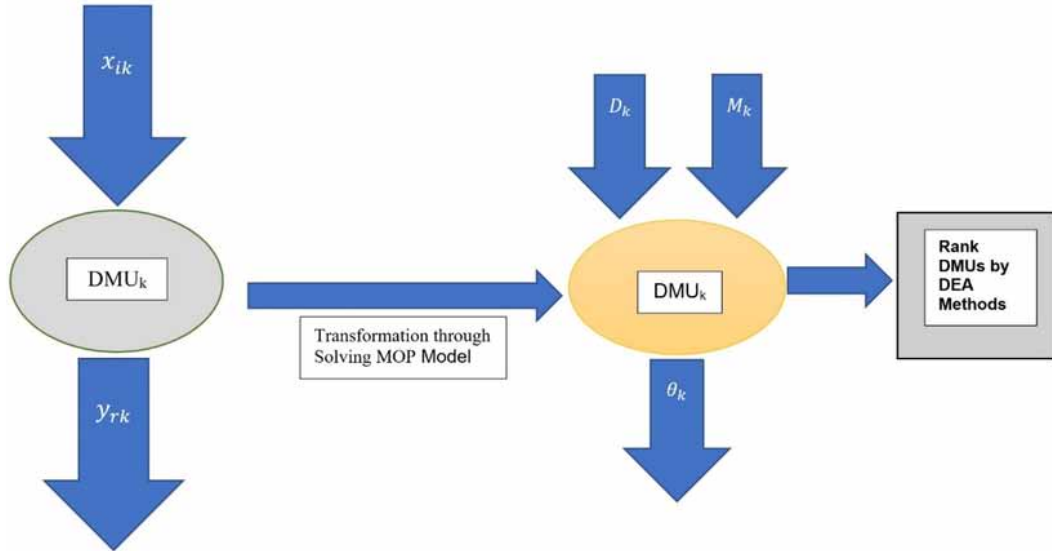
Note that the three performance measures for each DMU would be classified as one output, h_k , to be maximized and two inputs, M_k and D_k , to be minimized. Thus, each DMU, after solving the above MOP model for a given set of α , can be transformed into a new DMU with a single output, θ_k , and two inputs, D_k and M_k . Now, to apply DEA methods, there should be nonnegative relationships between the output, θ_k and the inputs, M_k and D_k , which is called an isotonicity condition. Consider the following Lemma (see Hong, 2020).

Lemma: For a specific efficient DMU_k, (i) the relationship between D_k in Eq. (6) and M_k in Eq. (5) is nonnegative, (ii) the relationship between D_k and θ_k in Eq. (4) is nonnegative, and (iii) the relationships between θ_k and M_k , and θ_k and D_k are nonnegative.

Proof: See Appendix 2.

The above **Lemma** satisfies the isotonicity condition (see Charnes and Cooper, 1985), so each DMU, after solving the above MOP model for a given set of α , can be transformed into a new DMU with one output, θ_k , and two inputs, M_k and D_k . Figure 1 depicts such a transformation of DMU with 's' outputs and 'm' inputs into a new DMU with a single output and two inputs. This process is called as the multiple-criteria with transformed DMUs (MCWTU) method. After all of the DMUs are transformed, various DEA methods could be applied to find the efficiency of these transformed DMUs. The transformed DMUs with two inputs and a single output is the simplest DMUs that any DEA-related methods start to be applied. Hence, the purpose of transforming the original DMU into a new DMU with a single output and two inputs is to increase the strengths of any DEA-related methods, regardless of the number of inputs and outputs that the original DMU has. Thus, with the

Figure 1. The Transformation process of DMUs through solving MOP model for multiple-criteria DEA



transformed DMUs have a single output and two outputs, each DEA method's weaknesses are expected to be minimized.

PROPOSED RANKING METHOD

Solving the MOP given in (13)-(17) for a given set value of α transforms a DMU with two inputs and a single output. Repeating it with different values of α would generate various alternative DMUs. Let N denote the total number of alternatives for a DMU after solving the MOP N times. Computing the average efficiency score, $\bar{\theta}_k = \sum_{n=1}^N \theta_{kn} / N$, is the first step, where θ_{kn} is the efficiency score of n^{th} configuration, $n=1, 2, \dots, N$. As Li and Reeves (1999) suggest, if the average efficiency scores of DMU_k is a perfect score of 1, DMU_k should be ranked as #1, and all other DMUs are ranked based on its own average efficiency score.

The second step is to apply several DEA methods for ranking DMUs to the transformed DMUs. The most popular standard ranking methods are cross efficiency, super-efficiency, and stratification methods. The CE method was suggested as a DEA extension to rank DMUs with the main idea of using DEA to do peer evaluation, rather than in pure self-evaluation. It consists of two phases. The first one is the self-evaluation phase (Phase I), where DEA scores are calculated using the model by (1)-(3). In the second phase (Phase II), the weights/multipliers arising from phase I are applied to all DMUs to get the cross-efficiency score (CES) for each of DMUs. In Phase I, let CE_{kk} represent the DEA score for DMU_k , the k^{th} option generated by the MOP model in (13) with (7)-(12) and (14)-(17), which is obtained from the following LP model:

$$\max CE_{kk} = u_k \theta_k \quad (18)$$

subject to:

$$v_{1k}D_k + v_{2k}M_k = 1 \quad (19)$$

$$u_k\theta_j - [v_{1k}D_j + v_{2k}M_j] \leq 0, \forall j \quad (20)$$

$$u_k, v_{ik} \geq 0, i = 1, 2$$

Now, the cross efficiency of DMU_j, using the weights that DMU_k has chosen in the model by (18)-(20), is given by:

$$CE_{kj} = \frac{u_k\theta_j}{[v_{1k}D_j + v_{2k}M_j]}, k \text{ and } j = 1, \dots, N, k \neq j \quad (21)$$

DMU_j is called a rated DMU, whereas DMU_k is called a rating DMU. Then, Doyle and Green (1994) use Eq. (21) to set up the CE matrix that consists of the self-evaluation value, CE_{kk} , in the leading diagonal and peer-evaluation value, CE_{kj} , in the non-diagonals. By averaging CE_{kj} in (21), the CES for DMU_k is defined as:

$$\overline{CE}_k = \frac{1}{N} \sum_{j=1}^N CE_{jk} \quad (22)$$

As mentioned before, especially the CE-DEA method of ranking the DMUs with many inputs and outputs shows very inconsistent results. But with transformed DMUs with just two inputs and a single output, the CE method would generate more robust or consistent rankings.

A super-efficiency (SE) DEA would generate a super-efficiency score (SES), which is obtained from the conventional DEA model after a DMU under evaluation is excluded in the reference set. In the SE method, the frontier line generated from the remaining DMUs changes for each efficient DMU to be evaluated, so the SESs of efficient DMUs can have higher values than 1, which is the maximum value in ES obtained by other DEA methods. The SE model, which has been applied significantly for ranking efficient DMUs, is given by:

$$\max SES_k = u_k\theta_k \quad (23)$$

subject to:

$$v_{1k}D_k + v_{2k}M_k = 1 \quad (24)$$

$$u_k\theta_j - [v_{1k}D_j + v_{2k}M_j] \leq 0, j \neq k \quad (25)$$

$$u_r, v_i \geq 0, r = 1, \dots, s; i = 1, \dots, m$$

Seiford and Zhu (2003) propose the stratification method in this context so that all DMUs are classified into several levels. Level 1 consists of all efficient DMUs found by the conventional DEA method. Then, after removing the DMUs in level 1, the DMUs that are found to be efficient are classified and belong to level 2, and so on. DMUs are stratified into different efficiency levels. Let $J^1 = \{DMU_j, j = 1, 2, \dots, N\}$ be the whole set of DMUs. Then, define $J^{\ell+1} = J^\ell - E^\ell$, iteratively, until $J^{\ell+1}$ becomes empty. E^ℓ consists of all the efficient DMUs on the ℓ^{th} level, that is, $E^\ell = \{DMU_k \in J^\ell \mid \sigma^*(\ell, k) = 1\}$, and $\sigma^*(\ell, k)$ is the optimal value to the following model for DMU_k under evaluation:

$$\sigma^*(\ell, k) = \min_{\lambda_j, \sigma(\ell, k)} \sigma(\ell, k) \quad (26)$$

subject to:

$$\sum_{j \in F(J^\ell)}^K \lambda_j (M_j + D_j) - \sigma(\ell, k) (M_k + D_k) \leq 0 \quad (27)$$

$$\sum_{j \in F(J^\ell)}^K \lambda_j \theta_j - \theta_k \geq 0 \quad (28)$$

$$\lambda_j \geq 0, j = 1, \dots, n$$

where $j \in F(J^\ell)$ means $DMU_j \in J^\ell$. The ES of the DMUs in the ℓ^{th} level is equal to 1 if all DMUs in the all the higher levels are removed from the whole set of DMUs. The DEA stratification model given by (26)-(28) partitions the set of DMUs into different frontier levels characterized by E^ℓ . The attractiveness score for each DMU in the ℓ^{th} stratification (E^ℓ) is computed against DMUs in the $(\ell + 1)^{th}$ and lower levels as the evaluation context (Zhu, 2014).

Procedure

Step 1: [Stratifying DMUs into levels]

- (i) Setting $\ell = 1$, let J^ℓ be the whole set of DMUs.
- (ii) Using the DEA method, evaluate all DMUs in J^ℓ by solving an LP given in (1)-(3).
- (iii) Identifying efficient DMUs where their ESs are equal to 1, stratify them into a set Θ^ℓ .
- (iv) Let $J^{\ell+1} = J^\ell - \Theta^\ell$ and set $\ell = \ell + 1$.
- (v) If $J^\ell = \emptyset$, set $\tau = \ell - 1$, and go to Step 2. Otherwise, go to (ii).

Step 2: [Transforming DMUs by solving MCONVENTIONAL DEA model using MOP model]

- (i) Set $\ell = 1$ and let $\Psi^\ell = \Theta^\ell$.
- (ii) Set various values of the weight set, $\alpha = \{\alpha_1^-, \alpha_2^+, \alpha_3^+\}$, each weight changes with an increment of Δ and between 0 and 1.
- (iii) Solve the C-DEA model with the objective function in (13) subject to constraints (7)-(12) and (14)-(17) for each given weight set for the DMU set, Ψ^ℓ .
- (iv) Compute $\bar{\theta}_k$, where $k \in \Psi^\ell$.

Step 3: [Computing ESs for the transformed DMUs]

- (i) Obtain the efficiency scores (ESs) using CE and SE models for the transformed DMUs with one output and two inputs, which are generated in (ii) of Step 2.
- (ii) Set $R_\ell(j)$ to be the rank of DMUj based on the values of ESs found in (i).
- (iii) Let $\Psi^{\ell+1} = \Psi^\ell + \Theta^{\ell+1}$ and set $\ell = \ell + 1$. If $\ell \leq \tau$, go to Step 2-(ii). Otherwise, calculate

$$\text{the average rank, } \bar{R}_\ell(j) = \frac{\sum_{\ell=1}^{\tau} R_\ell(j)}{\tau} \text{ and rank the DMUj in } \Theta^\ell.$$

NUMERICAL EXAMPLE

To investigate the performance of MCWTU-DEA methods, the data of Zhu (2014, p. 21), which are presented in Table 2, is used. In Table 2, there are fifteen (15) companies from the Top Fortune Global 500 list in 1995, three inputs: (i) assets (\$ millions), (ii) equity (\$ millions), and (iii) number of employees, and two outputs: (i) revenue (\$ millions) and (ii) profit (\$ millions). The stratification DEA (S-DEA) classifies all 15 DMUs into four levels, as shown in Table 3

As shown in Table 3, there are seven DMUs in Level 1, three in Level 2, two in Level 3, and three in Level 4. As expected, all seven DMUs in Level 1, 'Mitsui,' 'Itochu,' 'General Motors,' 'Sumitomo,' 'Exxon,' 'Wal-Mart,' and 'Nippon Life,' have an efficiency score (ES) of 1.000 by conventional DEA. The DMUs at a higher level have lower CESs and rankings than the DMUs of a lower level, as both CE method and MCWTU-based methods rank several DMUs in Level 1 lower than the DMUs of the lower levels. There are three different top-ranked DMUs. These are DMU #5, 'Sumitomo,' DMU #9, 'Exxon,' and DMU #13, 'Nippon Life,' which are ranked by CE, all MCWTU-based DEA methods, and SE method, respectively. The SE DEA is the only method that ranks DMUs following the levels generated by the stratification DEA. But the notable result is that the top two ranked DMUs, 'Nippon Life' and 'Wal-Mart,' by SE method, are ranked very low by some other methods. Note that the lowest ranks of 'Nippon Life' and 'Wal-Mart' are #13 and #14 by the CE method and MCWTU-based methods. These rankings are lower than even the rankings of some DMUs in Level 2, 3, and 4. These results would tend to make the ranking power of SE-DEA questionable. It is also observed that DMU #15, 'AT&T,' is bottom-ranked by all methods. Now the question is which company is really a top-ranked DMU, 'Sumitomo' or 'Exxon.'

Now, after removing DMU #12, DMU #14, and DMU #15 of Level 4, all methods are applied, and the results are reported in Table 4 for the DMUs of Level 1, 2, and 3. Note that the rankings generated by the SE method are not affected by removing the DMUs in Level 4. Regarding the second rank, two MCWTU-based methods consistently select 'General Motors' by CES and SES, while the CE method chooses 'Sumitomo.' To further see the behavior of rankings, with the DMUs in Level 3 and 4 removed, apply all methods for DMUs of Level 1 and 2 only, and the results are reported in Table 5. A notable observation is that CE-DEA changes the top-ranked DMU from 'Sumitomo' to 'Exxon,' which all MCWTU-based DEA methods consistently rank as No. 1. In fact, except for the SE method, all methods identify DMU #9, 'Exxon,' as the top-ranked DMU. SE DEA method does not change any single ranking. Finally, only the DMUs in Level 1 are considered, and the results are

Table 2. Fifteen (15) Companies from Fortune Global 500 Companies list in 1995

DMU	Company	Input			Output	
		Assets	Equity	Employees	Revenue	Profit
1	Mitsubishi	91,920.6	10,950.0	36000	184,365.2	346.2
2	Mitsui	68,770.9	5,553.9	80000	181,518.7	314.8
3	Itochu	65,708.9	4,271.1	7182	169,164.6	121.2
4	General Motors	217,123.4	23,345.5	709000	168,828.6	6,880.7
5	Sumitomo	50,268.9	6,681.0	6193	167,530.7	210.5
6	Marubeni	71,439.3	5,239.1	6702	161,057.4	156.6
7	Ford Motor	243,283.0	24,547.0	346990	137,137.0	4139
8	Toyota Motor	106,004.2	49,691.6	146855	111,052.0	2662.4
9	Exxon	91,296.0	40,436.0	82000	110,009.0	6470.0
10	Royal Dutch/Shell	118,011.6	58,986.4	104000	109,833.7	6904.6
11	Wal-Mart	37,871.0	14,762.0	675000	93,627.0	2740.0
12	Hitachi	91,620.9	29,907.2	331852	84,167.1	1468.8
13	Nippon Life	364,762.5	2,241.9	89690	83,206.7	2426.6
14	Nippon T & T	127,077.3	42,240.1	231400	81,937.2	2209.1
15	AT&T	88,884.0	17,274.0	299300	79,609.0	139.0

reported in Table 6. Surprisingly, the SE method finally ranks ‘Exxon’ as the top DMU and ‘Nippon Life’ as #2.

As the procedure suggests, the average rankings are computed for the efficient DMUs in Level 1 for each method and rank them based on the average rankings. To compare the regular DEAs (R-DEA: conventional, CE, and SE methods) with the proposed MCWTU-based DEA methods, the averages of average rankings are computed and reported in Table 7. Both methods identify the same DMUs, ‘Exxon,’ as the top-ranked one, and ‘Nippon Life’ as the #6 ranked DMU. For many decision-makers, identifying the top-level rated DMU among many evaluated DMUs would be an essential thing for them to make the final decision. For the top-level DMUs, applying CE-DEA or SE-DEA would mislead the decision-makers, as shown in this numerical example. CE and SE methods initially select ‘Sumito’ and ‘Nippon Life’ as the top-ranked DMU out of all 15 DMUs. As the inefficient DMUs of the lower levels from all DMUs to be evaluated are eliminated, these two R-DEA methods eventually choose ‘Exxon’ as the most efficient DMU. In contrast, the MCWTU-based CE and SE methods consistently identify ‘Exxon’ as the top DMU whether inefficient DMUs are included in the DMUs to be rated. An additional advantage for MCWTU-based methods comes from the observation that two generated rankings are almost identical, whereas R-DEA methods produce quite different rankings.

SUMMARY AND CONCLUSION

Several problems have appeared as the conventional DEA (C-DEA) has been applied to a wide variety of evaluation areas. The C-DEA evaluates DMUs in terms of self-evaluation, which allows each DMU to rate its efficiency score with the most favorable weights to itself. Consequently, the problems related to weak discriminating power have arisen as the applications of C-DEA advance, since multiple DMUs frequently turn out to be efficient with ES of 1. Many authors point out the lack of discrimination power as the major weakness for C-DEA. To remedy this weakness and increase

Table 3. Ranking table for DMUs at all Levels.

Level	DMU	Company	Efficiency Score		Cross Efficiency Score		Super Efficiency Score	
			Conventional DEA [R]	MCWTU [R]	CE DEA [R]	MCWTU [R]	SE DEA [R]	MCWTU [R]
1	2	Mitsui	1.0000 [1]	0.9116 [3]	0.6239 [7]	0.2459 [8]	1.0092 [7]	0.2737 [8]
	3	Itochu	1.0000 [1]	0.8182 [5]	0.8470 [3]	0.1905 [12]	1.2856 [6]	0.1999 [12]
	4	General Motors	1.0000 [1]	0.8913 [4]	0.5366 [8]	0.8705 [2]	1.3704 [3]	1.0046 [2]
	5	Sumitomo	1.0000 [1]	1.0000 [1]	0.9312 [1]	0.2412 [10]	1.3188 [5]	0.2561 [9]
	9	Exxon	1.0000 [1]	1.0000 [1]	0.8987 [2]	0.9345 [1]	1.3416 [4]	1.1829 [1]
	11	Wal-Mart	1.0000 [1]	0.1635 [14]	0.6383 [6]	0.2273 [14]	1.3792 [2]	0.1410 [14]
	13	Nippon Life	1.0000 [1]	0.2286 [13]	0.4091 [10]	0.2458 [9]	3.9176 [1]	0.2389 [11]
2	6	Marubeni	0.9719 [8]	0.7229 [7]	0.7689 [4]	0.1748 [13]	0.9719 [8]	0.1832 [13]
	7	Ford Motor	0.7372 [10]	0.5886 [9]	0.3820 [11]	0.5743 [4]	0.7372 [10]	0.6530 [4]
	10	Royal Dutch/shell	0.8414 [9]	0.7454 [6]	0.7049 [5]	0.8684 [3]	0.8414 [9]	0.9479 [3]
3	1	Mitsubishi	0.6628 [11]	0.6587 [8]	0.5044 [9]	0.2784 [7]	0.6628 [11]	0.2958 [7]
	8	Toyota Motor	0.5246 [12]	0.3882 [10]	0.3668 [12]	0.4067 [5]	0.5246 [12]	0.4246 [5]
4	12	Hitachi	0.3861 [13]	0.2885 [12]	0.2585 [14]	0.2273 [11]	0.3861 [13]	0.2437 [10]
	14	Nippon T & T	0.3486 [14]	0.3322 [11]	0.2600 [13]	0.3239 [6]	0.3486 [14]	0.3525 [6]
	15	AT&T	0.2704 [15]	0.1367 [15]	0.1523 [15]	0.0778 [15]	0.2704 [15]	0.0809 [15]

[R]: Ranking

the discrimination power, the cross-efficiency (CE) evaluation method, and the super-efficiency (SE) DEA, and the multiple criteria (MC) DEA method have emerged. As described before, these methods have also exhibited some weaknesses; especially, when the DMUs to be evaluated have many inputs to use and many outputs to produce.

This paper proposes an innovative procedure of ranking the DMUs in the DEA context by transforming the original DMUs with multiple inputs and outputs into new DMUs with two inputs and a single output, the simplest DMUs. The proposed procedure is called as the multiple-criteria with transformed DMUs (MCWTU) method. With these transformed DMUs, three regular DEA (R-DMU) methods are applied to see the performance of the MCWTU method. Using the stratification DEA, all DMUs are classified into multiple levels, depending on the efficiency scores, where the DMUs in Level 1 have a perfect ES of 1. After removing the DMUs in Level 1, the DMUs with ES of 1 belong to Level2, and so on.

To demonstrate the proposed procedure, this paper uses the well-known examples. The S-DEA is applied to classify all 15 DMUs, which generates four levels. In the first round, R-DEA methods, along with MCWTU-based DEA methods, are applied to evaluate all DMUs in four levels. Then, after removing the DMUs in the lowest level, Level 4, the DMUs of Level 1 through Level 3 are evaluated in the second round, and this process is continued. It is observed that all methods eventually end up identifying the same DMU as the most efficient one, while MCWTU-based DEA methods identify the most efficient one from the beginning. R-DEA methods identify a different DMU as a top-ranked one when all DMUs are evaluated and tend to detect the most efficient one as inefficient DMUs are removed from evaluation. Especially, SE DEA turns out to be the only method that ranks

Table 4. Ranking table for DMUs in Level 1, 2, and 3

Level	DMU	Company	Efficiency Score		Cross Efficiency Score		Super Efficiency Score	
			Conventional DEA [R]	MCWTU [R]	CE DEA [R]	MCWTU [R]	SE DEA [R]	MCWTU [R]
1	2	Mitsui	1.0000 [1]	0.9213 [4]	0.5672 [6]	0.2479 [7]	1.0092 [7]	0.2850 [7]
	3	Itochu	1.0000 [1]	0.9674 [3]	0.8505 [3]	0.2314 [9]	1.2856 [6]	0.2502 [9]
	4	General Motors	1.0000 [1]	0.8884 [5]	0.5177 [8]	0.8901 [2]	1.3704 [3]	1.0376 [2]
	5	Sumitomo	1.0000 [1]	1.0000 [1]	0.9140 [1]	0.2406 [8]	1.3188 [5]	0.2604 [8]
	9	Exxon	1.0000 [1]	1.0000 [1]	0.9051 [2]	0.9291 [1]	1.3416 [4]	1.1207 [1]
	11	Wal-Mart	1.0000 [1]	0.2105 [9]	0.5479 [7]	0.1526 [12]	1.3792 [2]	0.1820 [12]
	13	Nippon Life	1.0000 [1]	0.2452 [10]	0.4584 [10]	0.2085 [11]	3.9176 [1]	0.2105 [11]
2	6	Marubeni	0.9719 [8]	0.8408 [6]	0.7789 [4]	0.2159 [10]	0.9719 [8]	0.2334 [10]
	7	Ford Motor	0.7371 [10]	0.6132 [8]	0.3798 [11]	0.5284 [4]	0.7372 [10]	0.6406 [4]
	10	Royal Dutch/ Shell	0.8414 [9]	0.7464 [7]	0.7105 [5]	0.8094 [3]	0.8414 [9]	0.9519 [3]
3	1	Mitsubishi	0.6628 [11]	0.6589 [8]	0.4741 [9]	0.2776 [6]	0.6628 [11]	0.3003 [6]
	8	Toyota Motor	0.5246 [12]	0.3918 [10]	0.3469 [12]	0.3707 [5]	0.5246 [12]	0.4244 [5]

[R]: Ranking

Table 5. Ranking table for DMUs in Level 1 and 2

Level	DMU	Company	Efficiency Score		Cross Efficiency Score		Super Efficiency Score	
			Conventional DEA [R]	MCWTU [R]	CE DEA [R]	MCWTU [R]	SE DEA [R]	MCWTU [R]
1	2	Mitsui	1.0000 [1]	0.8768 [5]	0.6325 [7]	0.1993 [7]	1.0092 [7]	0.2054 [7]
	3	Itochu	1.0000 [1]	0.9826 [2]	0.8733 [3]	0.1741 [9]	1.2856 [6]	0.1827 [8]
	4	General Motors	1.0000 [1]	0.9677 [3]	0.6432 [6]	0.8995 [2]	1.3704 [3]	1.1225 [2]
	5	Sumitomo	1.0000 [1]	0.8984 [4]	0.8858 [2]	0.1761 [8]	1.3188 [5]	0.1820 [9]
	9	Exxon	1.0000 [1]	1.0000 [1]	0.9876 [1]	0.9062 [1]	1.3416 [4]	1.2698 [1]
	11	Wal-Mart	1.0000 [1]	0.6182 [9]	0.5676 [9]	0.4243 [5]	1.3792 [2]	0.4290 [5]
	13	Nippon Life	1.0000 [1]	0.4798 [10]	0.6072 [8]	0.3159 [6]	3.9176 [1]	0.3304 [6]
2	6	Marubeni	0.9719 [8]	0.8654 [6]	0.7830 [4]	0.1702 [10]	0.9719 [8]	0.1789 [10]
	7	Ford Motor	0.7371 [10]	0.7172 [8]	0.4769 [10]	0.5843 [4]	0.7372 [10]	0.6607 [4]
	10	Royal Dutch/ Shell	0.8414 [9]	0.7459 [7]	0.7657 [5]	0.7649 [3]	0.8414 [9]	0.9265 [3]

[R]: Ranking

Table 6. Ranking table for DMUs in Level 1.

Level	DMU	Company	Efficiency Score		Cross Efficiency Score		Super Efficiency Score	
			Conventional DEA [R]	MCWTU [R]	CE DEA [R]	MCWTU [R]	SE DEA [R]	MCWTU [R]
1	2	Mitsui	1.0000 [1]	0.8774 [4]	0.6657 [6]	0.1923 [5]	1.0092 [7]	0.1963 [5]
	3	Itochu	1.0000 [1]	1.0000 [1]	0.8066 [3]	0.1606 [7]	1.5531 [3]	0.1665 [7]
	4	General Motors	1.0000 [1]	1.0000 [1]	0.7322 [4]	0.9218 [2]	1.3704 [5]	1.1138 [2]
	5	Sumitomo	1.0000 [1]	0.8762 [5]	0.8409 [2]	0.1680 [6]	1.3252 [6]	0.1712 [6]
	9	Exxon	1.0000 [1]	0.9648 [3]	0.9823 [1]	0.9236 [1]	5.8757 [1]	1.1179 [1]
	11	Wal-Mart	1.0000 [1]	0.6760 [6]	0.7000 [5]	0.4236 [3]	1.3792 [4]	0.4298 [3]
	13	Nippon Life	1.0000 [1]	0.5563 [7]	0.5995 [7]	0.2778 [4]	3.9176 [2]	0.2957 [4]

[R]: Ranking

Table 7. Ranking table for average rankings of efficient DMUs in Level 1

Level	DMU	Company	Efficiency Score		Cross Efficiency Score		Super Efficiency Score		Average	
			Conventional DEA [R]	MCWTU [R]	CE DEA [R]	MCWTU [R]	SE DEA [R]	MCWTU [R]	R-DEAs [R]	MCWTU [R]
1	2	Mitsui	1.0 [1]	4.0 [5]	6.5 [4]	6.7 [4]	7.0 [7]	6.7 [3]	4.8 [7]	5.8 [5]
	3	Itochu	1.0 [1]	2.7 [2]	3.0 [3]	9.2 [7]	5.2 [5]	9.0 [7]	3.0 [3]	6.9 [4]
	4	General Motors	1.0 [1]	3.2 [4]	6.5 [4]	2.0 [2]	3.5 [3]	2.0 [2]	3.6 [5]	2.4 [2]
	5	Sumitomo	1.0 [1]	2.7 [2]	1.5 [1]	8.0 [3]	5.2 [5]	8.0 [5]	2.5 [2]	6.2 [3]
	9	Exxon	1.0 [1]	1.5 [1]	1.5 [1]	1.0 [1]	4.0 [4]	1.0 [1]	2.1 [1]	1.1 [1]
	11	Wal-Mart	1.0 [1]	9.5 [6]	6.7 [6]	8.5 [6]	2.5 [2]	8.5 [6]	3.4 [4]	8.8 [7]
	13	Nippon Life	1.0 [1]	10.0 [7]	8.7 [7]	7.5 [5]	1.2 [1]	7.0 [4]	3.6 [6]	8.5 [6]

[R]: Ranking, R-DEAs: Regular DEAs, Conventional, CE, and SE DEA.

the DMUs following the levels that the stratification DEA generates. But the top-ranked DMU by SE method is ranked very low by all other methods, including CE method, another kind of R-DEA method, until only efficient DMUs of Level 1 are evaluated. It shows that the proposed approach would be used as an important tool for decision-makers to identify the top-rated DMUs that the R-DEA methods may miss.

Various ranking methods, as shown in Table 1, other than R-DEA methods considered in this study, could be applied for the transformed DMUs created by modeling MC DEA as the goal programming. For future research, it would be motivating to verify if it is worthwhile and adequate to transform the DMUs to rank the original DMUs effectively.

REFERENCES

- Adler, N., Friedman, L., & Sinuany-Stern, Z. (2002). Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research*, 140(2), 249–265. doi:10.1016/S0377-2217(02)00068-1
- Aldamak, A., & Zolfaghari, S. (2017). Review of efficiency ranking methods in data envelopment analysis. *Measurement*, 106, 161–172. doi:10.1016/j.measurement.2017.04.028
- Anderson, A., & Petersen, C. N. (1993). A procedure for ranking efficient units in DEA. *Management Science*, 39(1), 1261–1264. doi:10.1287/mnsc.39.10.1261
- Anderson, T. R., Hollingsworth, K., & Inman, L. (2002). The fixed weighting nature of a cross-evaluation model. *Journal of Productivity Analysis*, 17(3), 249–255. doi:10.1023/A:1015012121760
- Cao, L., Ma, Z., & Muren, . (2020). Cooperation and competition strategy analysis of decision-making units based on efficiency game. *Journal of Systems Science and Systems Engineering*, 29(2), 235–248. doi:10.1007/s11518-019-5417-9
- Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research*, 2(6), 429–444. doi:10.1016/0377-2217(78)90138-8
- Charnes, A., Haag, S., Jaska, P., & Semple, J. (1992). Sensitivity of efficiency classifications in the additive model of data envelopment analysis. *International Journal of Systems Science*, 23(5), 789–798. doi:10.1080/00207729208949248
- Chen, Y., Cook, W. D., & Lim, S. (2019). Preface: DEA and its applications in operations and data analytics. *Annals of Operations Research*, 278(1-2), 1–4. doi:10.1007/s10479-019-03243-w
- Chu, J., Wu, J., & Liu, M. (2020). A new DEA common-weight multi-criteria decision-making approach for technology selection. *International Journal of Production Research*, 58(2), 3686–3700. doi:10.1080/00207543.2019.1634294
- Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA)- Thirty years on. *European Journal of Operational Research*, 192(1), 1–17. doi:10.1016/j.ejor.2008.01.032
- Deng, Z., Gao, Y., Liang, B., & Morrison, A. M. (2018). Efficiency evaluation of hotel operations in Mainland China based on the super efficiency SBM model. *Tourism Economics*, 26(2), 276–298. doi:10.1177/1354816618816787
- Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *The Journal of the Operational Research Society*, 45(5), 567–578. doi:10.1057/jors.1994.84
- Gavгани, S.S., & Zohrehbandian, M. (2014). A cross-efficiency based ranking method for finding the most efficient DMU. *Mathematical Problems in Engineering*.
- Guo, I., Lee, H., & Lee, D. (2017). An integrated model for slack-based measure of super-efficiency in additive DEA. *Omega*, 67, 160–167. doi:10.1016/j.omega.2016.05.002
- Hong, J. D. (2020). Transforming decision making units in the DEA context. Working Paper 20-007, South Carolina State University, Orangeburg, SC, USA.
- Hong, J. D., & Jeong, K. Y. (2017). Cross-efficiency based heuristics to rank decision making units in data envelopment analysis. *Computers & Industrial Engineering*, 111, 320–330. doi:10.1016/j.cie.2017.06.015
- Hou, Q., Wang, M., & Zhou, X. (2018). Improved DEA cross efficiency evaluation method based on ideal and anti-ideal point. *Discrete Dynamics in Nature and Science*. doi:10.1155/2018/1604298
- Jahanshahloo, G. R., Lotfi, H. F., Sanei, M., & Jelodar, M. F. (2008). Review of ranking models in data envelopment analysis. *Applied Mathematical Sciences*, 2(29), 1431–1448.
- Jiang, C., & Lin, W. (2015). DEA Rank: A data-envelopment-analysis-based ranking method. *Machine Learning*, 101(1-3), 415–435. doi:10.1007/s10994-014-5442-3
- Li, X. B., & Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research*, 115(3), 507–517. doi:10.1016/S0377-2217(98)00130-1

- Liang, L., Wu, J., Cook, W., & Zhu, J. (2008). The DEA game cross efficiency model and its nash equilibrium. *Operations Research*, 56(5), 1278–1288. doi:10.1287/opre.1070.0487
- Liu, H. H., Song, Y. Y., & Yang, G. L. (2019). Cross-efficiency evaluation in data envelopment analysis based on prospect theory. *European Journal of Operational Research*, 273(1), 364–375. doi:10.1016/j.ejor.2018.07.046
- Lotfi, H. F., Jahanshahloo, G. R., Khodabakhshi, M., Rostamy-Malihlifeh, M., Moghaddas, Z., & Vaez-Ghasemi, M. (2013). A review of ranking models in data envelopment analysis. *Journal of Applied Mathematics*, 2013, 492421.
- Markovits-Somogy, R. (2011). Ranking efficient and inefficient decision-making units in data envelopment analysis. *IJTTE. International Journal for Traffic and Transport Engineering*, 1(4), 245–256.
- Nayebi, M., & Lotfi, F. H. (2016). The methods of ranking based super efficiency. *International Journal of Research in Industrial Engineering*, 5(4), 43–66.
- Ragsdale, C. T. (2017). *Spreadsheet modeling & decision analysis: A practical introduction to business analytics* (8th ed.). Cengage Learning.
- Ramanathan, R. (2006). Data envelopment analysis for weight derivation and aggregation in the analytic hierarchy process. *Computers & Operations Research*, 33(5), 1289–1307. doi:10.1016/j.cor.2004.09.020
- Seiford, L. M., & Zhu, J. (2003). Context-dependent data envelopment analysis measuring attractiveness and progress. *Omega*, 31(5), 397–408. doi:10.1016/S0305-0483(03)00080-X
- Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. In R. Silkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis* (pp. 73–105). Jossey-Bass.
- Shahghobadi, S. (2020). Utilization of performance indicators in data envelopment analysis to increase the efficiency discrimination of units. *Computers & Industrial Engineering*, 145, 106535. doi:10.1016/j.cie.2020.106535
- Toloo, M., Keshavarz, E., & Hatami-Marbini, A. (2020). (in press). Selecting data envelopment analysis models: A data-driven application to EU countries. *Omega*. Advance online publication. doi:10.1016/j.omega.2020.102248
- Tran, T. H., Mao, Y., Nathanail, P., Siebers, P., & Robinson, D. (2019). Integrating slacks-based measure of efficiency and super-efficiency in data envelopment analysis. *Omega*, 85, 156–165. doi:10.1016/j.omega.2018.06.008
- Wang, Y. M., & Chin, K. S. (2010). A neutral DEA model for cross-efficiency evaluation and its extension. *Expert Systems with Applications*, 37(5), 3666–3675. doi:10.1016/j.eswa.2009.10.024
- Zhu, J. (2014). *Quantitative models for performance evaluation and benchmarking: data envelopment analysis with spreadsheets* (3rd ed.). Springer.

APPENDIX 1

Acronyms

- **C-DEA:** Conventional Data Envelopment Analysis
- **CE:** Cross Efficiency
- **CES:** Cross Efficiency Score
- **CRS:** Constant Returns to Scale
- **DEA:** Data Envelopment Analysis
- **DMU:** Decision Making Unit
- **ES:** Efficient Score
- **LP: Linear Programming**
- **MC:** Multiple Criteria
- **MCWTU:** Multiple Criteria with Transformed DMU
- **MOP:** Multi-Objective Programming
- **R-DEA:** Regular Data Envelopment Analysis
- **SE:** Super Efficiency
- **SES:** Super Efficiency Score

APPENDIX 2

Proofs of Lemma

- (i) Set $D_k \left(= \sum_j d_j \right)$ equal to some constant. If $d_w = \text{Max}\{d_j\}$ increases, both D_k and $M_k \left(= \text{Max}\{d_j\} \right)$ increase. If $d_w \neq \text{Max}\{d_j\}$ increases, $M_k \left(= \text{Max}\{d_j\} \right)$ does not decrease, while D_k increases. Thus, the fact that as D_k increases, M_k does not decrease completes the proof.
- (ii) If DMU_k is efficient, it implies that $\theta_k = 1$. That is:

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (1)$$

and:

$$\sum_{r=1}^s u_r y_{rj} = 1 \quad (2)$$

From Eq. (8), d_j can be expressed as:

$$d_j = \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj}, \quad \forall j \quad (3)$$

Let the current sum of the slack variables $D_k^c = \sum_j d_j$. Now, for DMU_h , $h \neq k$, to reduce d_h , the first term of (B.3), $\sum_{i=1}^m v_i x_{ih}$, should be reduced, subject to the constraint, $\sum_{i=1}^m v_i x_{ik} = 1$. Let v_i' and u_r' denote the revised weights for inputs and outputs to reduce D_k^c , respectively. From (B.1), v_i' and u_r' must satisfy the following constraints:

$$\sum_{i=1}^m v_i' x_{ik} = 1 \quad (4)$$

$$\sum_{i=1}^m v_i' x_{ih} \leq \sum_{i=1}^m v_i x_{ih} \quad (5)$$

Since (B.3) should be nonnegative:

$$\sum_{r=1}^s u_r y_{rh} \leq \sum_{i=1}^m v_i x_{ih} \quad (6)$$

Thus, if $\sum_{r=1}^s u_r y_{rj} \leq \sum_{r=1}^s u_r y_{rj}$, D_k is further reduced, (B.6.) is rewritten as:

$$\sum_{r=1}^s u_r y_{rj} \leq \sum_{i=1}^m v_i x_{ij} \quad (7)$$

From (B.7), the revised efficiency score, $\theta'_k = \sum_{r=1}^s u_r y_{rk} / \sum_{i=1}^m v_i x_{ik}$, can't be greater than

$\theta_k = \sum_{r=1}^s u_r y_{rk} / \sum_{i=1}^m v_i x_{ik}$. The fact that θ_k can't increase as D_k decreases completes the proof.

(iii) Due to the nonnegative relationship between D_k and θ_k and the nonnegative relationship between D_k and M_k , the fact that the relationship between θ_k and M_k is nonnegative completes the proof.