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Stochastic Modelling of Weather-Related Transmission Line Outages

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ABSTRACT

The physical environment around transmission lines plays a major role in the resulting reliability of the power network. The inclusion of weather in the failure and repair process will lead to realistic modelling of the power network. This article suggests a modelling methodology to take into account weather-related failures. Besides a maintenance management strategy using dynamic programming, it is suggested to minimizing the cost of maintenance while accounting for weather-related failures. The data obtained from 220kV Transmission lines from Goa, India, is used to stochastically model the phenomenon. A three-state weather model is suggested, and accordingly the failure and repair phenomenon are segregated and stochastically modelled. Time-varying expressions for computing the availability in each weather condition is computed. This model can be used by the power utilities to realistically model weather-related failures.

KEYWORDS

Condition Based Maintenance, Maintenance Management, Markov Model, Power System, Power Utility, Reliability of Power Systems, Stochastic Process, Transmission Line Outage

1. INTRODUCTION

In India, the unavailability of power based on weather is a cause of outrage and bitter criticism from the public against the power utilities. For example, during the rainy season, the frequency of power failure and down time is high leading to protest against the Government and power utilities (Gauree, 2018; Saini, 2018; Sony & Mekoth, 2014, 2015). Among the other reasons of power failure, the weather is one of the most important factors affecting the reliability of power systems (Panteli, Pickering, Wilkinson, Dawson, & Mancarella, 2017; Zhou, Pahwa, & Yang, 2006). The supply chain model of the power system from Generation, Transmission, and Distribution has to work together to cater the reliable power to consumers (Pabla, 2012). The transmission and distribution lines are affected by weather-related failures. Analysis of transmission line is important because the failure of transmission lines will lead to the power outage of several distribution lines leading to blackouts (Sony Michael & Mariappan, 2012). The failure rates are dependent on the weather conditions around which the transmission line operates. Increased failure rates are reported in different types of weather condition (Wu, Gao, Tang, & Huang, 2016).

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Not only the failure rates the downtime are also influenced by the weather conditions (Tavner et al., 2013). For an overhead transmission system, the weather environment can severely impact the performance of the transmission lines leading to the power utility's operational ability due to enhanced line failure and repair rates during bad weather periods (Billinton & Singh, 2006). Therefore, estimating reliability by utilities without incorporating weather conditions can be quite optimistic and erroneous. In addition, it may also affect planning and design decisions for maintenance activities for the power utilities, which will result in the financial loss (Fanucchi, Bessani, Camillo, London, & Maciel, 2016). By recognizing the impact of various weather contributions to the total system performance indices will help to pinpoint the situations, where investment may be provided for maximum reliability improvement. A transmission and distribution system usually consists of overhead lines, underground and combination of both (Pabla, 2012). In utilities like India, which is a developing country, the ratio of overhead transmission lines to underground transmission lines are very high (Saini, 2018). Hence, assessing the reliability of systems without incorporating, weather-related outages would lead to erroneous conclusions resulting in bad policy and maintenance strategies. In other words, for cash-starved utilities, incorporating weather-related outages will lead to optimum allocation of available resources. Another aspect to consider in a developing country is that the repair process is also dependent on weather (Sony Michael, Mariappan, & Kamat, 2011). Thus, failure rate and repair rate are a continuous function of weather conditions. It is difficult to describe failure rate and repair rate as a continuous function of weather-related failures. However, a set of discrete states could be used to describe the variable failure and repair rates. Previous researchers have used the concept of failure bunching to model storm related failures (Billinton & Singh, 2006; Gaver, Montmeat, & Patton, 1964). Markov models were used in the evaluation of transmission system availability if the rate of failure and repair followed exponential distribution (Michael et al., 2009).

Besides failure modelling of storm-related failure were also modelled as a two-state process (Billinton & Bollinger,1968). A three-state weather process model was proposed by Billington and Singh (2006) to account for weather-related faults. However, this model cannot be used when repair process is weather-related. Nevertheless, due to the practical impact of weather-related failure on repair rates, it makes the problem, further complex in a developing country power utility and the three-state model will not be able to capture its complexity (Michael, Amonkar, Mariappan, & Kamat, 2009). In addition, classification of failure rates into monsoon, winter, and summer, further improves the modelling complexity and actuates the system to its reality. In this paper, it is proposed to stochastically model the weather-related outages in a transmission line. A mathematical model is proposed for its evaluation and a case study is presented on the failure of the 220kV Transmission system in Goa, India. In addition, a maintenance strategy for taking into account weather-related failure as also suggested. The model can be used readily by power utilities for evaluating its availability and maintenance strategy designed.

2. WEATHER RELATED FAILURE-REPAIR MODEL

Modelling failure related to weather conditions like the heavy storm, freezing rains, tornados etc. can have a great impact on the reliability of power systems (Billinton & Acharya, 2006; Trakas, Hatziargyriou, Panteli, & Mancarella, 2016). The IEEE Standard 346 further suggests that the weather environment should be divided into the three classes of normal, adverse and major storm disaster (Wang & Billinton, 2002). As such we propose a weather based three- state model. In order to model first state, we classify the weather during Summer. For modelling adverse state, we consider the weather during winter where fog deposits on the insulators are created failures (Gorur, Cherney, & Hackam, 1986). To model major storm disaster, we model the weather as Monsoon (Liu, 2015).

In addition, it is further proposed from our practical discussion with power utility Engineers in India, that repair process is also affected by these three weather conditions. To ascertain this fact, three focus group discussion (Morgan, 1998) was conducted. A thematic conclusion which transpired from the discussion was that repair rate was dependent on the weather. Thus, the repair rate was classified as repair during summer, winter and rainy. The state transition diagram is given in Figure 1. The state 1 is the state during which the transmission line is working. State 2 is depicted as a failure rate (x1) in Summer and accordingly its corresponding repair rate(u1). The state 3 is shown as a failure rate of line during winter (x2) and with its repair rate (u2). State 4 is denoted as a failure rate (x3) during monsoon with its repair rate (u3).

3. FAILURE AND REPAIR DATA ANALYSIS

The failure and repair data analysis of a 220kV transmission line from Goa Electricity Department was conducted for last five years. The utility under consideration is a state-run power utility. The fault database of the 220kV line was searched. The Time between failure and repair was accordingly calculated for each data point using Microsoft Excel. Monsoon-related data was categorized from June to September, Winter related data was categorized from October to January and Summer data from February to May. The three-category data was tested for distribution and Anderson darling test was performed suggesting that the failure and repair data confirmed to exponential distribution. Similarly, the repair data was analyzed, and it was found that it also conformed to exponential distribution. The probability plot for Summer, winter and Monsoon failure along with parameter estimated is presented in Figure 2.

The hazard rate is calculated and plotted in Figure 3. The probability plot for summer, winter and monsoon repair is computed and further plotted in Figure 4. The hazard rate for summer, winter and monsoon is also plotted in Figure 5. In addition, the Anderson Darling test was conducted to ascertain that data follows the exponential process. The repair process of this line consisted for physical patrolling the line to locate the exact location of the fault. These transmission lines pass through dense forest and hence, vehicular access is ruled out leading to physical patrolling of the line by employees of the

Figure 1. State transition diagram

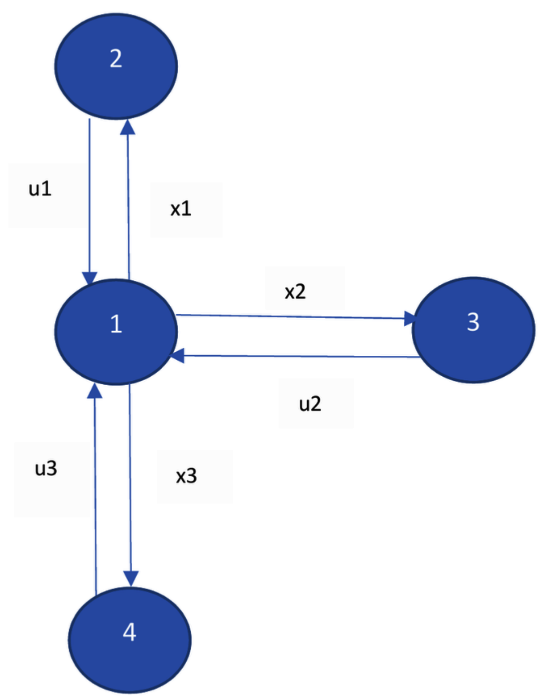


Figure 2. Probability plot of Summer, Winter and monsoon failure

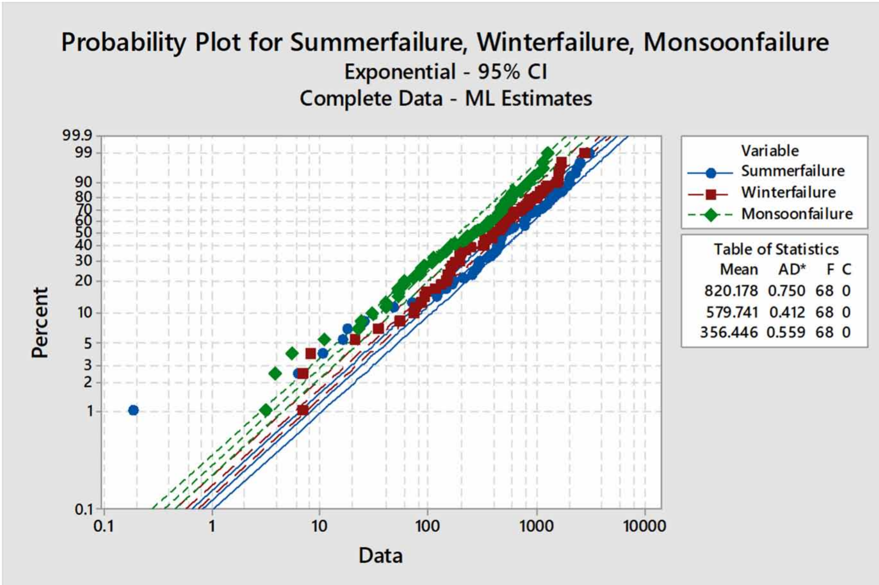
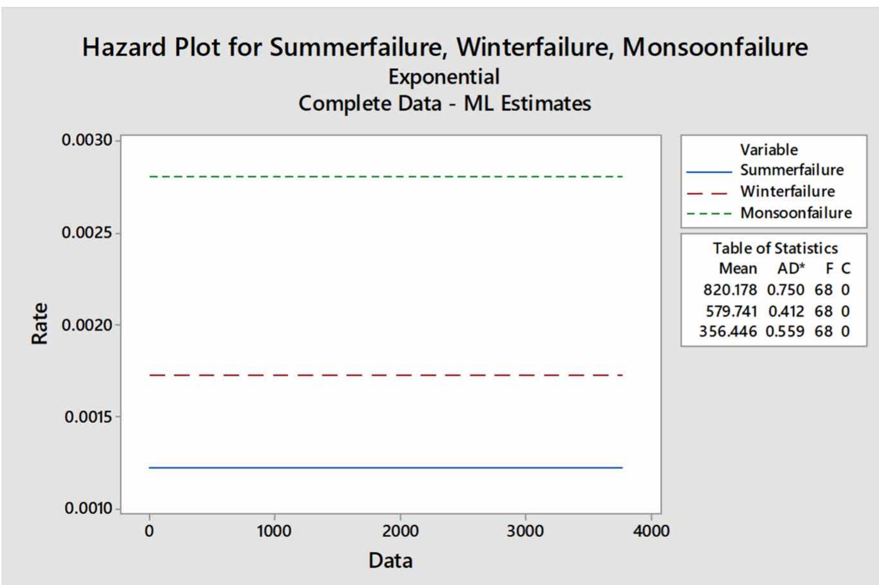


Figure 3. Hazard rate of Summer, Winter and monsoon failure



power utility and further attending of the fault. In order to ascertain that statistically, the parameters are statistically different, an ANOVA was carried out on the failure and repair rate parameters. It is ascertained that the failure rates are indeed different for summer, winter. The Dunnett's Multiple comparison was done, and results are shown in Figure 6, Table 1 and Table 2. Similarly, these tests were done on the repair rates and results are shown in Figure 7, Table 2 and Table 3.

Figure 4. Probability plot for repair rates

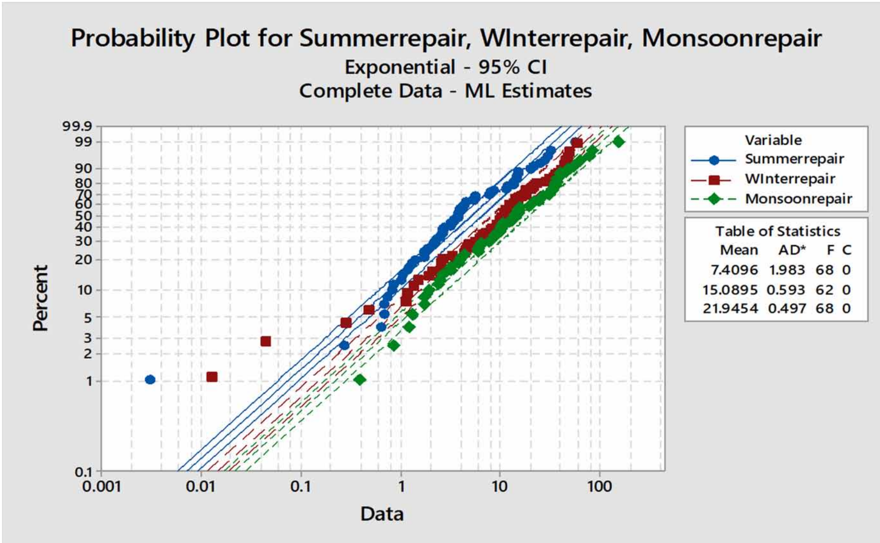
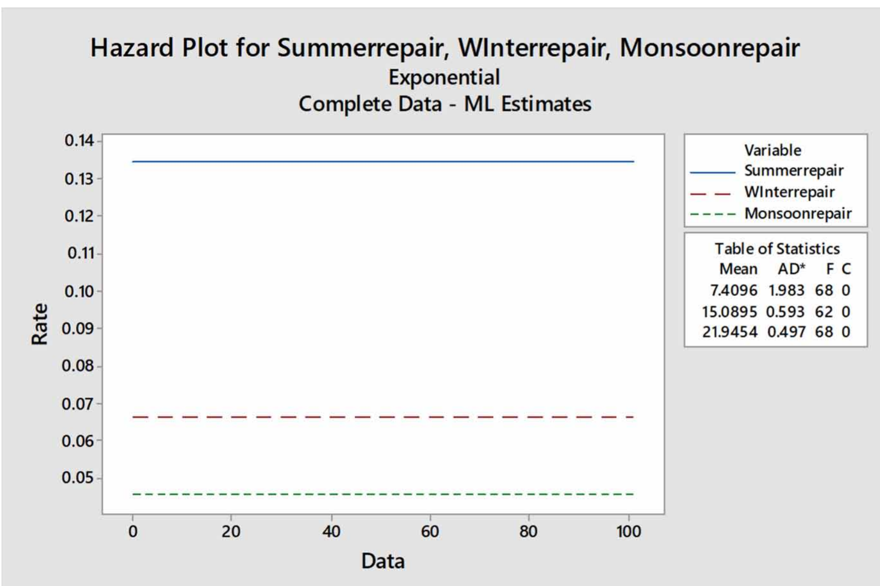


Figure 5. Hazard rate for repair rates



4. MARKOV PROCESS

The exponential failure and repair rates point to modelling the entire state space as a Markov process. Till today in the power systems context, continuous parameter Markov models have been applied most extensively to model power system reliability and maintenance problems (Chan & Asgarpoor, 2006; Michael et al., 2009). Hence, the transmission line is assumed to be repairable. The time to failure and repair depends on the weather condition such as summer, winter, and monsoon. The failure – repair process is modelled as occurring in a limited number of discrete steps.

Figure 6. Dunnets simultaneous plot

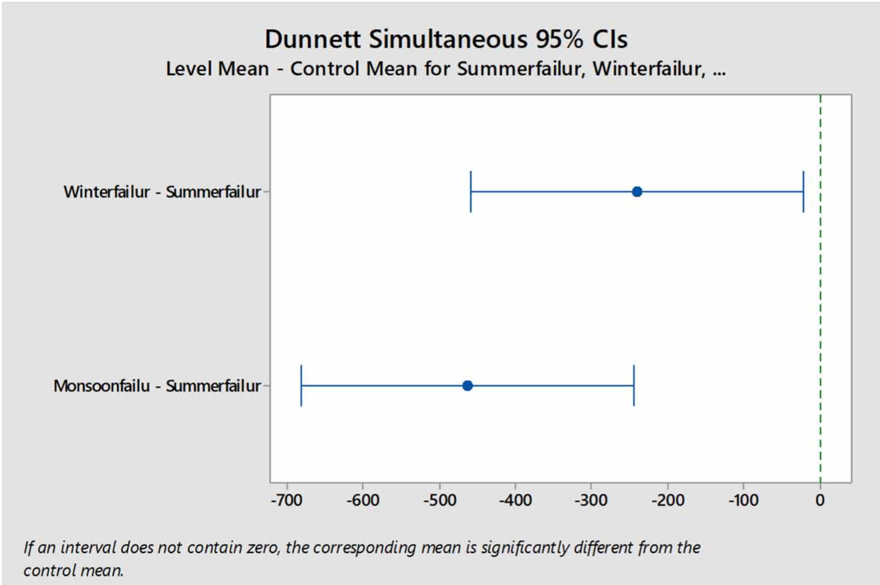


Table 1. Analysis of variance failure rates

Source DF Adj SS Adj MS F-Value P-Value
Factor 2 7314945 3657472 11.22 0.000
Error 201 65512804 325934
Total 203 72827749

Table 2. Dunnett multiple comparisons with a control

Grouping Information Using the Dunnett Method and 95% Confidence
Factor N Mean Grouping
Summerfailure (control) 68 820.2 A
Winterfailure 68 579.7
Monsoonfailure 68 356.4
Means not labeled with the letter A are significantly different from the control level mean.

Notation:

x1 –failure rate of line in summer
x2- failure rate of line in winter
x3- failure rate of line in monsoon
u1 – repair rate of line in Summer
u2 – repair rate of line in winter
u3 – repair rate of line in monsoon

Figure 7. Dunnets simultaneous plot

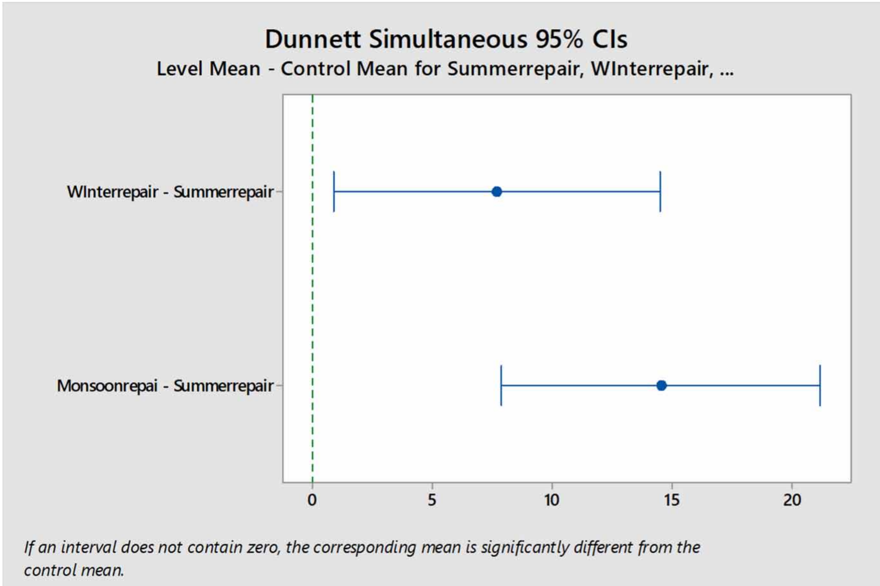


Table 3. Analysis of variance

Source DF Adj SS Adj MS F-Value P-Value
Factor 2 7191 3595.5 11.88 0.000
Error 195 59034 302.7
Total 197 66225

Table 4. Dunnett multiple comparisons with a control

Grouping Information Using the Dunnett Method and 95% Confidence
Factor N Mean Grouping
Summerrepair (control) 68 7.41 A
Monsoonrepair 68 21.95
WInterrepair 62 15.09
Means not labeled with the letter A are significantly different from the control level mean.

Table 5. State table

State	1	2	3	4
Line	Working	Summer related Failure	Winter related Failure	Monsoon related Failure

$P_1(t)$ = probability that the system is in state one at t
$P_2(t)$ = probability that the system is in state two at t
$P_3(t)$ = probability that the system is in state three at t
$P_4(t)$ = probability that the system is in state four at t

$$\begin{bmatrix} S+x1+x2+x3 & -u1 & -u2 & -u3 \\ -x1 & S+u1 & 0 & 0 \\ -x2 & 0 & S+u2 & 0 \\ -x3 & 0 & 0 & S+u3 \end{bmatrix} x \begin{bmatrix} P1\left(t\right) \\ P2\left(t\right) \\ P3\left(t\right) \\ P4\left(t\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the same using lower triangular Matrix and then further taking inverse Laplace transforms we have four-time dependent equations. Substituting and solving the same for steady state conditions. The steady state probability of the system is elucidated are, the probability that the power system is working $P1(\infty) = 0.9118$. The probability of failure in Summer is $P2(\infty) = 0.0082$. The probability of failure during winter $P2(\infty) = 0.0238$ and the probability of failure in monsoon is 0.0562.

4.1. Cycle Time

By making the process S-independent, the cycle time between encountering of states is computed as the ratio(Alsammarae, 1989), hence it is calculated for summer= $\frac{1}{A2.\left(x2\right)}$, for winter $= \frac{1}{A3.\left(x3\right)}$ and for monsoon it is calculated as= $\frac{1}{A4x\left(x4\right)}$. The cycle time for Summer is 899.52 hours, Winter is 631.21 and monsoon 390.64 hrs.

5. MAINTENANCE STRATEGY DESIGN

The maintenance strategy could be designed by taking into consideration the cycle time of each state. In order to design the maintenance strategy during the summer, the cycle time expectancy is around 899.53 hours which is around 37.4 days. The maintenance strategy should revolve around this time period. During the winter season, the maintenance strategy should revolve around 631.2 hours which is around 26.3 days. The monsoon season has a cycle time of around 390.64 hours or 16.27 days. By accounting for the weather-related failure pattern, the maintenance strategy for each season is continuously changing. It is a well-accepted rule that Condition-based maintenance (CBM) is suitable for exponential failures patterns except where failure consequences will not result in revenue losses, customer dissatisfaction or health, safety and environmental impact (Andrawus et al., 2007). Thus, a CBM model with exponential failures and fixed inspection intervals considering the cycle should be very effective (Barbera et al., 1996). There are many predictive maintenance methods out of which the five techniques most often used are:

1. Vibration Analysis
2. Motor Current Analysis
3. Ultrasonic Pulse-Echo Techniques
4. Tribology
5. Infrared Thermography

It is further suggested that the condition of the transmission lines is monitored within the cycle time for each weather condition. It could be noticed that the variable indicating the condition if above the threshold, an immediate resort to maintenance activity could have resorted too at the earliest. The transmission lines under consideration can fail, after the cycle time. The failure pattern follows exponential distribution. As such the failure rate would also be dependent on the condition. An important aspect to take into consideration would be to minimize the long run average cost of maintenance and failures. However, failure of transmission lines causes loss not only to the utility, but also to the society. Hence minimizing long run average cost of maintenance calls for a condition-based maintenance models for exponential failure and fixed inspection interval based on cycle time would be most ideal. The following conceptual groundings is elucidated. Let the condition of the transmission line be described by a quantity a_t. This quantity a_t measures any of the five predictive maintenance methods earlier suggested as a continuous variable. It is further assumed from the data that time to failure follows the exponential process. The failure rate, in this case, would be a function of $\lambda(a_t)$. In order, to prove our point we assume that the failure occurs after the midpoint of the cycle time. The deterioration of the transmission line is modelled as b_t. This occurs during the period as an independent and further it is identically distributed non-negative random variable. This is exponentially distributed in this case. It is during this period, that on the transmission line, a preventive action should be performed or not. In order to model the costs, fixed cost 't_k' is charged for preventive maintenance. The failure cost 't_r' is charged for failure of the transmission line. It is assumed that $t_r > t_k$.

After the preventive maintenance, an assumption is made that the transmission line is restored to the original condition or initial value a_o. This is a fixed decision parameter. When there occurs no failure, during the period 't', the measurement of a_{t+1} at the beginning of the period t+1:

$$a_{t+1} = a_t + b_t \tag{1}$$

In order to simplify, each state is considered i.e. reliability of single operating and failed state. The probability, that the system will not fail till the end of the time period 't' is given by the expression $e^{-\lambda(a_t+b_t)T}$. The cost of failure during the inspection period is $t_r\left(1-e^{-\lambda(a_t+b_t)T}\right)$. Thus, the conditional expected cost of failure is $\int_0^\infty t_r\left(1-e^{-\lambda(a_t+b_t)T}\right).f\left(b\right)db.$ After the preventive maintenance activities, the transmission line is returned to from the state a_t to the initial state a_o. Hence the decision variable in this case would be $d(a_t) = 0$ for no maintenance and 1 for maintenance action.

Further the condition is described by the variable $a_t{*}{=}a_t-\left[a_t-a_0\right]d\left(a_t\right)$. To find the threshold condition a_t for the preventive maintenance of the transmission line, it is proposed to use dynamic programming.

The cost $D_t(a_t)$ = minimum cost of period t. In addition to this the future cost of period t+1 would be $D_{t+1}(X_{t+1})$. To minimize the current cost in the period t, is $t_k.D\left(a_t\right)+C\left(a_t*\right)+D_{t+1}(a_t*+\text{b})+(D_{t+1}\left(a_0\right)$.

Thus, the future cost to be minimized boils down to:

$$Min\ \{t_k.D\left(a_t\right)+C\left(a_t*\right)+\int_0^\infty D_{t+1}\left(a_t*+b\right).e^{-\lambda(a_t*+b)T}.f\left(b\right)db+D_{t+1}.\left(a_0\right)\left[1-\int_0^\infty e^{-\lambda(a_t*+b)T}f\left(b\right).db\right]$$

This expression can be solved numerically. To solve this expression dynamically two things, have to be assumed. First the smallest measurement unit of deterioration Δ and second would be to use Poisson distribution to model multiples of deterioration. This should be solved for each state to minimize the costs.

6. EFFECT OF MAINTENANCE STRATEGY DESIGN

Transmission line is run-repair-run system. By classifying the failures that occur in a transmission line into meaningful categories, an accurate modelling of the system is possible. In this case, the study was conducted in India, and a three-state model summer, winter and monsoon were envisaged. Thus, after solving the model we will have the probability of residing and transiting from each state. In addition, the cycle time for each state can be calculated. This data is used in the design of preventative maintenance strategy even though the system in this case is an exponential system, because the failure consequence is very high. The effect of this strategy would be high-availability of the transmission system in all three weather. This is because, there is a preventative maintenance strategy in all three-state weather solution for optimum reliability-based inspection would be different for each weather. In addition, the specific solution also considers the repair cost dynamics. Power utilities can use this approach to design specific tailor-made maintenance strategies for each situation.

7. CONCLUSION

To calculate the reliability of transmission lines one of the important factors to account for is the weather. This paper incorporates a methodology for calculating weather related failure and repairs rate was also modelled on this concept. Time-varying availability expression was computed by approximating the failure and repair phenomenon during different weather condition as a Markov Process. Besides a maintenance management strategy were suggested by considering the cycle time between the states using dynamic programming. The future work should concentrate on semi-Markovian or non-Markovian processes for failure or repair phenomenon.

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APPENDIX

Equations

P1(t) =
(u1^2*u2^2*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u3^2*x2*sum(exp(r3*t)/ (2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2^2*u3^2*x1*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2*x3*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3*x2*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2^2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3^2*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3*x1*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3^2*x1*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3))

+ u1^2*u2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u3*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2^2*u3*x1*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)))/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1) + (u1*u2*u3)/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1)

P2(t) =
(u2*u3*x1)/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1) - (u2^2*u3^2*x1*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3*x1*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3^2*x1*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2^2*u3*x1*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u2^2*x1*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u3^2*x1*x2*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 +s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 +s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3^2*x1*x2*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2

+ 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2^2*u3*x1*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u2*x1*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x 1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u3*x1*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3*x1*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u2*u3*x1*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)))/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1)

P3(t) =
(u1*u3*x2)/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1) - (u1^2*u3^2*x2*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3*x2*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3^2*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u3*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3^2*x1*x2*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3

+ u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u2*u3^2*x1*x2*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1^2*u2*x2*x3*sum(exp(r3*t)/ (2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u3*x2*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3*x1*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u2*x2*x3*sum((r3*exp(r3*t))/ (2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u2*u3*x1*x2*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x 1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u3*x2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3))))/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1)

P4(t)=
(u1*u2*x3)/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1) - (u1^2*u2^2*x3*sum(exp(r3*t)/ (2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2*x3*sum((r3^2*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) +

u1*u2^2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2^2*x1*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1^2*u2*x2*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1^2*u3*x2*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u2^2*u3*x1*x3*sum(exp(r3*t)/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2*x1*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) + u1*u2*x2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u1*u3*x2*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)) - u2*u3*x1*x3*sum((r3*exp(r3*t))/(2*r3*u1 + 2*r3*u2 + 2*r3*u3 + u1*u2 + 2*r3*x1 + u1*u3 + 2*r3*x2 + u2*u3 + 2*r3*x3 + u1*x2 + u2*x1 + u1*x3 + u3*x1 + u2*x3 + u3*x2 + 3*r3^2), r3 in RootOf(s3^3 + s3^2*x3 + s3^2*x2 + s3^2*x1 + s3^2*u3 + s3^2*u2 + s3^2*u1 + s3*u3*x2 + s3*u2*x3 + s3*u3*x1 + s3*u1*x3 + s3*u2*x1 + s3*u1*x2 + s3*u2*u3 + s3*u1*u3 + s3*u1*u2 + u2*u3*x1 + u1*u3*x2 + u1*u2*x3 + u1*u2*u3, s3)))/(u1*u2*u3 + u1*u2*x3 + u1*u3*x2 + u2*u3*x1)